

科目：工程數學甲【通訊聯招甲組、電機系甲、丁、戊、庚組、通訊所乙組；電機系丙組選考】

1. (15%) Solve the differential equation

$$\frac{dy_1}{dx} + 3y_1 + 3\frac{dy_2}{dx} = 2e^{-2x} \quad \text{with } y_1(0) = y_2(0) = 0.$$

$$\frac{dy_2}{dx} + 2y_2 - y_1 = 1$$

2. (15%) Use the power series method to solve the following equation

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

where n is a nonnegative integer.3. (20%) Let $L: V \rightarrow W$ be a one-to-one and onto, hence invertible, linear transformation between vector spaces V and W . Denote its inverse by L^{-1} , hence L^{-1} is a transformation from vector space W to vector space V .(a) Show that L^{-1} is also linear. (8%)(b) Let $\{z_1, \dots, z_r\}$ be a basis for W . Show that $\{L^{-1}(z_1), \dots, L^{-1}(z_r)\}$ is a basis for V . (12%)4. (14%) Given matrices $A \in \mathbb{R}^{m \times p}$ and $B \in \mathbb{R}^{m \times q}$, with A being full column rank, consider the linear equation $Ax + By = 0$. By choosing a vector $y \in \mathbb{R}^q$, we may consider to solve vector $x \in \mathbb{R}^p$ to satisfy the equation.

以下小題僅需依序寫下答案即可，不需做任何推導。

(a) What is the condition on y so that the equation is consistent, i.e. it is solvable? (4%)By knowing that every full column rank matrix has a left inverse, let $C \in \mathbb{R}^{p \times m}$ be such that $CA = I$.(b) If y is chosen so that the considered equation is consistent, what is the general form of the solution to the equation? (4%)If y is chosen so that the considered equation is inconsistent, then the concept of *projection* plays an important role in studying the solution of the equation. Let U and V be two subspaces of \mathbb{R}^m such that $\mathbb{R}^m = U \oplus V$ and let P and Q be the projection matrices that project vectors of \mathbb{R}^m onto U along V and onto V along U , respectively. It means that, $\forall r \in \mathbb{R}^m$, $Pr \in U$, $Qr \in V$, and $r = Pr + Qr$ is the unique decomposition associated with the two projections.(c) Let $U = R(A)$, the range of A , and let $V = N(C)$, the null space of C , what is P ? (6%)5. (36%) Each of the following problems has only one correct answer. You will obtain 3 points for each question if you give the correct answer. However, you will get -1 points for each question if the answer is not correct. Zero point will be deducted if you give no answer. The lowest point in this set of problems you may obtain is zero. For instance, if you give one correct answer, four wrong answers, and no answers for the rest of the problems, then you will obtain zero point for this problem set. Note that $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors in the positive direction of the Cartesian x, y, z coordinate system respectively. If $z = x + jy$ is a complex variable, then x, y are real numbers, and “ j ” denotes the pure imaginary number $(0,1)$.

第五大題請於答案卷「是非、選擇題專用區」作答。

【5.1 於該區第(1)格作答、5.2 於該區第(2)格作答……，依此類推】

科目：工程數學甲【通訊聯招甲組、電機系甲、丁、戊、庚組、通訊所乙組；電機系丙組選考】

5.1 Consider the following three systems, where $y[n]$ or $y(t)$ denotes the system output, and $x[n]$ or $x(t)$ is the system input:

$$\text{I. } y[n] = x[-n], \quad -\infty < n < \infty$$

$$\text{II. } y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$$

$$\text{III. } y(t) = [\sin(6t)]x(t)$$

Which of the following statements is correct?

- (a) I is time-invariant, II is linear, III is memoryless
- (b) I is causal, II is stable, III is memoryless
- (c) I is stable, II is linear, III is causal
- (d) I is memoryless, II is stable, III is linear
- (e) None of the above statements are correct

5.2 Suppose that the Fourier transform of the function $f(t) = te^{-4t^2}$ is $F(\omega) = ae^{-\omega^2/b}$, then which of the following statements is correct?

- (a) a is a real number, and $b > 20$
- (b) a is an imaginary number, and $14 < b < 18$
- (c) Both a and b are real numbers
- (d) $a = j\sqrt{\pi}\omega/c$, $c > 10$, $b < 20$
- (e) None of the above statements are correct

5.3 Let $f_1(t) = e^{\sin 3t}$, $f_2(t) = \sin |t|$, $f_3(t) = \sin(\sin t)$, $-\infty < t < \infty$. Which of the following statements is correct?

- (a) functions $f_1(t)$ and $f_2(t)$ are periodic, $f_3(t)$ is not periodic
- (b) functions $f_1(t)$ and $f_3(t)$ are periodic, $f_2(t)$ is not periodic
- (c) functions $f_1(t)$ and $f_2(t)$ are not periodic, $f_3(t)$ is periodic
- (d) functions $f_1(t)$, $f_2(t)$, and $f_3(t)$ are all periodic
- (e) None of the above statements are correct

5.4 Consider a second-order rational fraction

$$f(z) = \frac{1 - 1.7z^{-1}}{(1 - 0.8z^{-1})(1 - 1.25z^{-1})}$$

Suppose that the region of convergence is $0.8 < |z| < 1.25$, then the inverse z -transform of $f(z)$ is

- (a) $x[n] = 2(0.8)^n u[n] + (1.25)^n u[-n-1]$
- (b) $x[n] = \{2(0.8)^n - (1.25)^n\} u[n]$
- (c) $x[n] = 2(0.8)^n u[-n-1] + (1.25)^n u[n]$
- (d) $x[n] = \{2(0.8)^n - (1.25)^n\} u[-n-1]$
- (e) None of the above statements are correct

5.5 If one wants to compute the outflow per unit volume of a vector field $\mathbf{v} = (x^2/z)\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$ at a point $P = (2, 9, -1)$, which of the following is the correct way to do?

- (a) Compute the curl of \mathbf{v} at P , i.e., $\text{curl } \mathbf{v} \triangleq \nabla \times \mathbf{v} = -\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$
- (b) Compute the divergence of \mathbf{v} at P , i.e., $\text{div } \mathbf{v} \triangleq \nabla \cdot \mathbf{v} = 5$
- (c) Compute $\text{curl}(\text{curl of } \mathbf{v})$ at P , i.e., $\nabla \times (\nabla \times \mathbf{v}) = 8\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$
- (d) Compute the Laplacian of \mathbf{v} at P , i.e., $\nabla^2 \mathbf{v} = -10\hat{\mathbf{i}}$
- (e) None of the above methods are correct

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5.6 The directional derivative of a function $f(x, y, z) = x^2 + yz + xz^2$ in the direction of the vector $\mathbf{v} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ at a point $P = (1, 0, 1)$ is denoted as α . Then

- (a) $\alpha \leq 0$ (b) $4 \leq \alpha \leq 10$ (c) $14 \leq \alpha \leq 16$ (d) $0 < \alpha \leq 2$
 (e) None of the above answers are correct

5.7 Let a position vector be $\hat{\mathbf{P}}(\tau) = (\cos \tau)\hat{\mathbf{i}} + \sqrt{15}\tau\hat{\mathbf{j}} + (\sin \tau)\hat{\mathbf{k}}$. $\hat{\mathbf{P}}(\tau)$ will generate a curve as τ varies from a to b , i.e., $a \leq \tau \leq b$. Suppose that the length of the curve from $a = \pi/2$ to $b = 3\pi/2$ is L , then

- (a) $L = 2\pi$ (b) $L = 16\pi$ (c) $5\pi \leq L \leq 8\pi$ (d) $12\pi \leq L \leq 14\pi$
 (e) None of the above answers are correct

5.8 It is well known that the work done by an applied force along a specific trajectory can be computed by using line integral. Assume that the force is $\hat{\mathbf{F}}(x, y, z) = x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}} + z^2\hat{\mathbf{k}}$, and the trajectory is specified by $C: x = \cos t, y = \sin t, z = t, 0 \leq t \leq \pi$. Suppose that the work done by this force $\hat{\mathbf{F}}$ along the curve C is β/γ . Then

- (a) $20 \leq \beta \leq 25, 1 \leq \gamma \leq 5$ (b) $33 \leq \beta \leq 38, 6 \leq \gamma$ (c) $26 \leq \beta \leq 32, 1 \leq \gamma \leq 3$
 (d) $\beta \leq 25, 6 \leq \gamma \leq 10$ (e) None of the above answers are correct

5.9 Let $f(z) = (1-x)^3 + jy^3$, where $z = x + jy$ is a complex variable. We also let $f'(z) \triangleq df(z)/dz$. Which of the following statements is correct?

- (a) $f'(z)$ exists only at $z = 1$
 (b) $f'(z)$ exists only at $z = 0$
 (c) $f'(z)$ does not exist $\forall z$ in the complex plane
 (d) $f'(z)$ exists except at $z = 1$
 (e) None of the above statements are correct

5.10 Let $z = x + jy$ be a complex variable. Which of the following is an open set?

- (a) $|z + 1 - j| \leq 2$ (b) $\text{Im } z = 2$, where Im stands for imaginary part (c) $|z + 2 - j3| \geq 1$
 (d) $0 \leq \arg z \leq \pi/3, z \neq 0$, where "arg" stands for argument
 (e) None of the above sets are open

5.11 Let z be a complex number. Which of the following statements is correct?

- (a) $|\cos z|^2 \leq 1$
 (b) $\sin \bar{z}$ is analytic at $z = 0$, where \bar{z} is the complex conjugate of z
 (c) $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2) \quad \forall z_1$ and z_2 in the complex plane, where "Arg" stands for the principal value of $\arg z$
 (d) $|e^{z^2}| \leq e^{|z|^2}$
 (e) None of the above statements are correct

5.12 Compute the value of $\oint_C \tan z \, dz = \sigma + j\rho$, where C stands for the circle $|z| = 2$ in counterclockwise direction. Then which of the following statements is correct?

- (a) $\sigma = 0, \pi \leq \rho \leq 2\pi$
 (b) $\sigma = 0, \rho = 0$
 (c) $0 \leq \sigma \leq 2\pi, \rho = 0$
 (d) $\sigma = 0, -4\pi \leq \rho \leq -2\pi$
 (e) None of the above statements are correct

科目：電子學【通訊聯招甲組、電機系甲、乙、戊組、通訊所乙組】

- (10%) A voltage amplifier, when connected to a $10\text{k}\Omega$ source, has an overall gain of 1667 V/V . When a second identical amplifier is connected in parallel to the same source, the corresponding gain for each is found to be 909 V/V . Find the input resistance of the amplifier.
- (10%) What is the highest frequency of a triangle wave of 20-V peak-to-peak amplitude that can be reproduced by an op amp whose slew rate is $10\text{ V}/\mu\text{s}$? For a sine wave of the same frequency, what is the maximum amplitude of output signal that remains undistorted?
- (10%) For the circuit shown in Fig. 1, diodes D_1 through D_4 are identical. Each has $n=1$ and is a "1-mA diode". For small input signals, find the value of small-signal transmission v_o/v_i for $I=10\mu\text{A}$. If $I=1\text{mA}$, what is the largest possible output signal for which the diode currents deviate by at most 10% of their dc values?
- (10%) For a $0.8\text{ }\mu\text{m}$ CMOS fabrication process: $V_{tn}=0.8\text{V}$, $V_{tp}=-0.9\text{ V}$, $\mu_n C_{ox}=90\text{ }\mu\text{A}/\text{V}^2$, $\mu_p C_{ox}=30\text{ }\mu\text{A}/\text{V}^2$, $C_{ox}=1.9\text{ fF}/\mu\text{m}^2$, $\phi_f=0.34\text{ V}$, $\gamma=0.5\text{ V}^{1/2}$, V_A (n -channel devices) $=8L$ (μm), and $|V_A|$ (p -channel devices) $=12L$ (μm). Find the small-signal model parameters (g_m , r_o , and g_{mb}) for both an NMOS and a PMOS transistor having $W/L=20\text{ }\mu\text{m}/2\text{ }\mu\text{m}$ and operating at $I_D=100\text{ }\mu\text{A}$ with $|V_{SB}|=1\text{V}$. Also find the overdrive voltage at which each device must be operating.
- (10%) For the circuit shown in Fig. 2, v_{sig} is a small sine-wave signal with zero average. The transistor β is 100. (a) Find the value of R_E to establish a dc emitter current of about 0.5 mA . (b) Find R_C to establish a dc collector voltage of about $+5\text{V}$. (c) For $R_L=10\text{ k}\Omega$ and the transistor $r_o=200\text{ k}\Omega$, determine the overall voltage gain.
- (17%) From Fig. 3, $R_{B1}=50\text{ }\Omega$, $R_{C2}=15\text{ k}\Omega$, and $I_B=0.5\text{ mA}$. The transistors have $\beta=100$, $C_\mu=2\text{ pF}$, and $f_T=400\text{ MHz}$, neglect r_x and r_o . Calculate the midband gain A_M and the 3dB frequency f_H .
- (16%) From Fig. 4, $R_E=150\text{ }\Omega$, $R_C=7.5\text{ k}\Omega$, $I_B=1\text{ mA}$. Find the voltage gain and input resistance of the amplifier assuming that $\beta=150$.
- (17%) From Fig. 5, the circuit is a feedback circuit. The load resistance is R_L . Assume the op amp is ideal. (a) Identify the feedback topology. (b) Find the expression for A_f . (c) Find the expression for β .

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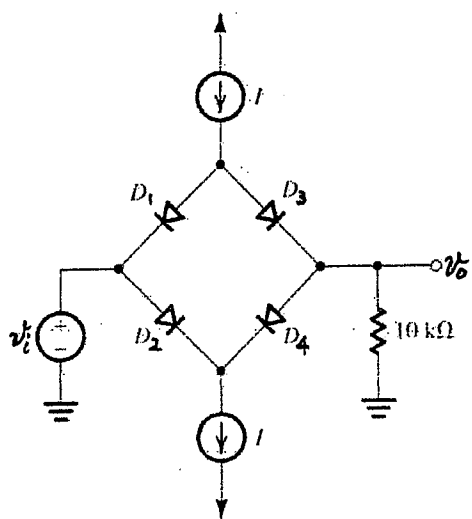


Figure 1

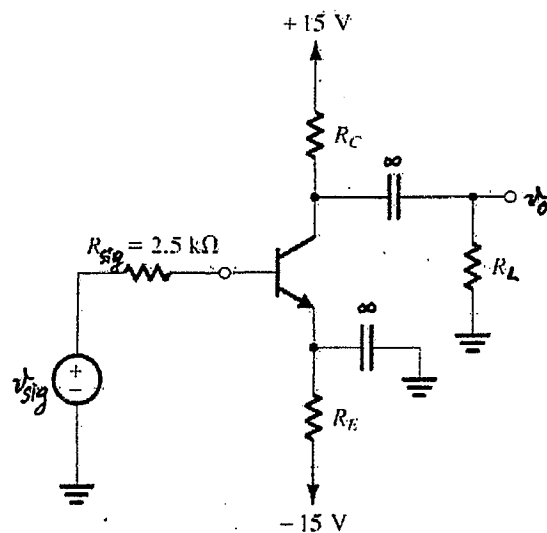


Figure 2

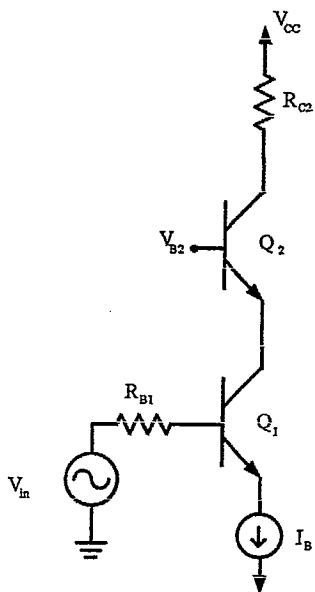


Figure 3

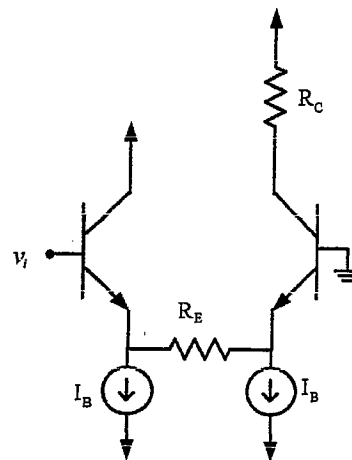


Figure 4

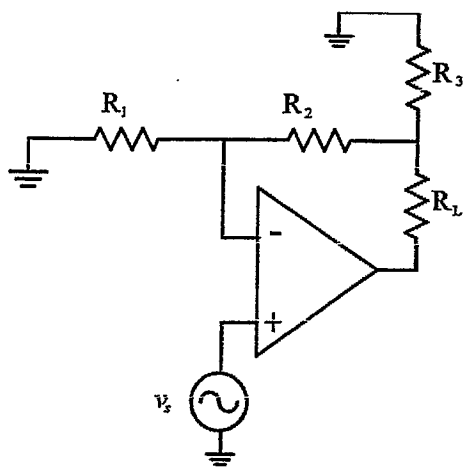
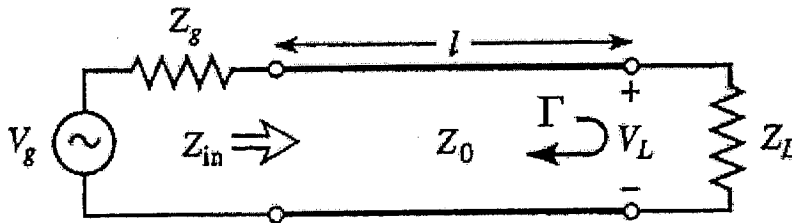


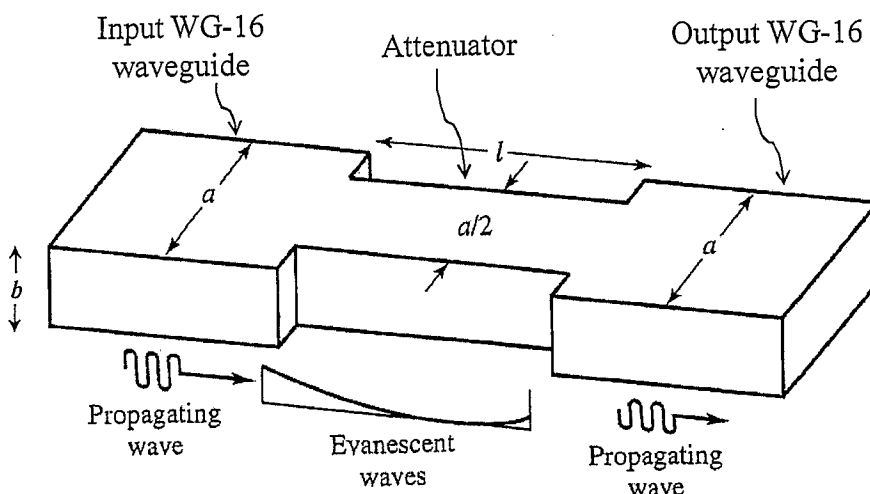
Figure 5

科目：電磁學【通訊聯招碩士班甲組、電機系碩士班戊組、通訊所碩士班乙組】✓

1. (25%) The inner conducting spherical shell of radius a is held at a potential V_1 and the outer shell of radius b at V_2 . The region between these two concentric shells is filled with an insulating material. Determine the potential distribution between the shells.
2. (25%) Determine the inductance per unit length of an air coaxial transmission line with a solid inner conductor of radius R_i and a very thin outer conductor of inner radius R_o . ($R_o/R_i = 5$ and $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$)
3. The transmission line circuit shown below has $V_g = 1 \angle 0^\circ$, $Z_g = 50 \Omega$, $Z_0 = 50 \Omega$, $Z_L = 30 - j20 \Omega$, and $l = 0.6 \lambda$. Calculate the power delivered to the load (denoted as P_L) using three different ways as follows.
 - (a) (10 %) Find the reflection coefficient (Γ) by which P_L can be further calculated.
 - (b) (10 %) Find the input impedance (Z_{in}) by which P_L can be further calculated.
 - (c) (10 %) Find the load voltage (V_L) by which P_L can be further calculated.



4. (a) (10%) A standard WG-16 air-filled rectangular waveguide for X-band applications has the dimensions: $a = 2.29 \text{ cm}$ and $b = 1.02 \text{ cm}$. If this WG-16 waveguide operates only in the dominant mode (TE_{10} mode), what is the allowable operating-frequency range?
- (b) (10%) An attenuator connected between the input and output WG-16 waveguides is made using a section of rectangular waveguide with a width of $a/2$ and a length of l , and this waveguide section operates below its dominant mode cutoff frequency. At the operating frequency of 12 GHz, determine the required length, l , of this waveguide section to achieve an attenuation of 100 dB between the input and output WG-16 waveguides.



科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】 ✓

1. (10%) Consider a matrix A defined by its diagonalization as follows:

$$A = CDC^{-1} = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 & 0 \\ 4 & 3 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 6 & 5 \end{bmatrix}^{-1},$$

where the columns of matrix C are the eigenvectors of A . Find the eigenvalues and the corresponding eigenvectors of A^T .

2. (10%) Find the singular value decomposition of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$.

3. For the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(a) (5%) Find $2A^4 + 3A^3 - 41A^2 - 111A + 5I$.

(b) (5%) Find $|AA^T|$.

(c) (5%) Find $| (A^{-2})^T |$.

4. (15%) Suppose W_1 and W_2 are the subspace of V .

Prove that $V = W_1 \oplus W_2 \leftrightarrow \dim(V) = \dim(W_1 + W_2) = \dim(W_1) + \dim(W_2)$.

5. (15%) Please verify that the inverse matrix of $A+BCD$ equals to

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$$

科目：線性代數【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】

6. (20%) Given the following matrices:

(i)
$$\begin{bmatrix} 2 & i \\ -i & 1 \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 1+i & 3 \\ 3 & 5 \end{bmatrix}$$

(iii)
$$\begin{bmatrix} 3 & 1+i & 1 \\ 1-i & 2 & -i \\ 1 & i & 5 \end{bmatrix}$$

(iv)
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

- (a) (5%) Which are skew-symmetric?
(b) (5%) Which are Hermitian?
(c) (5%) Which are unitary?
(d) (5%) Which are positive definite?

7. (a) (5%) Apply the Gram-Schmidt process to the following vectors to form a set of orthonormal bases.

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

(b) (5%) Find the orthogonal projection of the vector $[2 \ 1 \ -3 \ 2]^T$ on the space spanned by u_1 , u_2 , and u_3 .

(c) (5%) Find the QR decomposition of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

科目：機率【通訊聯招碩士班乙組、電機系碩士班己組、通訊所碩士班甲組】✓

1. Random variables X and Y have respective density functions defined as

$$f_X(x) = \frac{1}{a}[u(x) - u(x-a)]$$

$$f_Y(y) = bu(y)e^{-by}$$

for $a > 0$ and $b > 0$, and $u(x)$ is denoted as the *unit step* function of x .

- (a) Find and sketch the density function of $Z = X + Y$ if X and Y are statistically independent. (7%)
- (b) What is the moment generating function of Z . (8%)
2. (*Linear amplifier with cutoff*) Let us consider X to be the random variable with zero-mean and unit variance Gaussian distribution, $N(0, 1)$, and consider a non-linear device with transmittance, $y = g(x)$, to be defined as

$$g(x) = 0, \quad |x| \geq 1$$

$$g(x) = x, \quad -1 < x < 1.$$

Given that

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$$

- (a) Please find the probability density function of $Y = g(X)$, $f_Y(y)$. (10%).
- (b) (*Infinite roots*) If we consider the *periodic extension* of the transmittance, that is the extended $g(x)$ is defined as

$$g(x) = \sum_{n=-\infty}^{\infty} (x - 2n) \text{rect}\left(\frac{x - 2n}{2}\right),$$

where

$$\text{rect}\left(\frac{x}{2}\right) = \begin{cases} 1, & |x| \leq 1 \\ 0, & \text{otherwise.} \end{cases} \quad (10\%)$$

Find $f_Y(y)$.

3. Let $Y = A \cos(\omega t + \Theta)$ where A, ω and t are constants, and Θ is a uniform random variable in the interval $(0, 2\pi)$. The random variable Y results from sampling the amplitude of a sinusoidal with random phase Θ . (a) Find $E[Y]$ (8%) and (b) $E[Y^2]$ (7%).
4. Let X be a random variable and $A = \{X \leq b\}$.
- (a) Find the conditional probability distribution function, $F_{X|A}(X|A)$ (10%).
- (b) What is the conditional density function $f_{X|A}(X|A)$? (5%)

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5. Let $Z = X_1 + X_2$ with $f_{X_1}(x)$, $f_{X_2}(x)$ and $f_Z(z)$ denoting the probability functions of X_1 , and X_2 , and Z , respectively.

(a) Show that $\Phi_Z(\omega) = \Phi_{X_1}(\omega)\Phi_{X_2}(\omega)$ (10%)

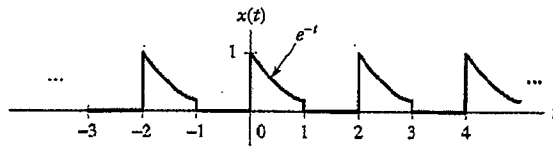
(b) If $f_{X_1}(x) = f_{X_2}(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\left(\frac{x - \mu_X}{2}\right)^2\right\}$, please find $\Phi_Z(\omega)$ and $f_Z(z)$ (10%).

6. Show that (a) $E[Y] = E[E[Y|X]]$ (7%). (b) $E[Z|X] = E[E[Z|X,Y]|X]$ (8%).

(Note: $E[Y] = \int_{-\infty}^{\infty} y f_Y(y) dy$)

通訊理論 (Communications Theory)

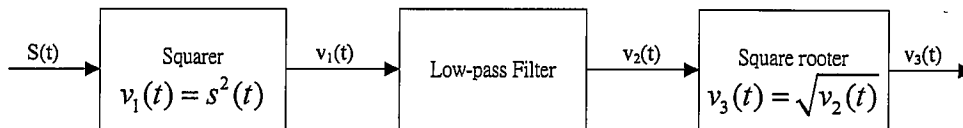
1. (15 points) A signal is given as follows.



- (a) (7 points) Explain whether the signal is an energy signal or a power signal.
- (b) (8 points) Find the Fourier representation of the signal.

2. (20 points) An AM signal $s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$ is considered in the following systems:

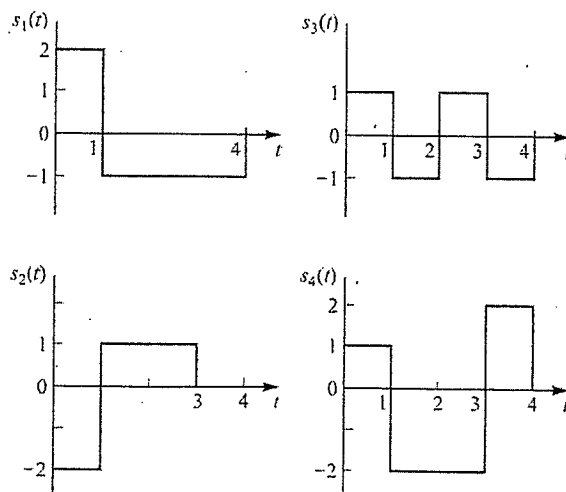
- (a) (10 points) If the signal $s(t)$ is used as input to a square-law detector whose transfer characteristic is defined by $v_0(t) = a_1 v_i(t) + a_2 v_i^2(t)$, where a_1 and a_2 are constants, $v_i(t)$ is the input, and $v_0(t)$ is the output. Find the conditions for which the message signal $m(t)$ may be recovered from $v_0(t)$.
- (b) (10 points) The signal $s(t)$ is applied to the system below. Assuming that $|k_a m(t)| < 1$ for all t and the message signal $m(t)$ is limited to the spectral interval $-W \leq f \leq W$ and that the carrier frequency $f_c > 2W$, show that $m(t)$ can be obtained from the square-rooter output $v_3(t)$.



3. (15 points) An FM signal with a frequency deviation of 10 kHz at a modulation frequency of 5 kHz is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiple triples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output.

4. (20 points) Gram-Schmidt Orthogonalization Procedure

- (a) (10 points) Suppose that we have a set of finite energy signal waveforms $\{s_i(t), i = 1, 2, \dots, M\}$. Please describe the Gram-Schmidt orthogonalization procedure to construct a set of orthonormal waveforms.
- (b) (5 points) Consider the four waveforms shown in the following figure. Determine the dimensionality of the waveforms and a set of orthonormal functions.
- (c) (5 points) From (b), use the orthonormal functions to represent the four waveforms by vectors $s_1, s_2, s_3,$ and s_4 .



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5. (20 points) Explanations

- (a) (5 points) Please describe the Nyquist sampling theorem for a band-limited signal of finite energy.
- (b) (5 points) Please describe the Nyquist criterion for distortionless baseband transmission in the absence of noise.
- (c) (5 points) Consider a random process $X(t)$ that is initiated at $t = -\infty$. Let $X(t_1), X(t_2), \dots, X(t_k)$ denote the random variables obtained by observing the random process $X(t)$ at times t_1, t_2, \dots, t_k , respectively. The joint distribution function of this set of random variables is $F_{X(t_1), X(t_2), \dots, X(t_k)}(x_1, x_2, \dots, x_k)$. Please describe the condition that the random process $X(t)$ is said to be stationary in the strict sense.
- (d) (5 points) Please describe the conditions that the process $X(t)$ is said to be Ergodic in the mean.

6. (10 points) Signal Constellation

For the QAM signal constellation shown in the following figure, determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.

