# 國立中山大學100學年度碩士班招生考試試題

科目：線性代數【通訊所碩士班甲組】

單選題 (6x5%=30%):

1. Consider a 3 × 3 matrix

$$A = \begin{bmatrix} 2 & -i & 1+i \\ i & 1 & 0 \\ 1-i & 0 & 1 \end{bmatrix},$$

[i] **A** is an Hermitian matrix

[ii] **A** is positive definite

[iii] The determinant of **A** is 1

[iv] The eigenvalues of **A** are 2, 1 and 1

[v] The trace of **A** is 4

Which statements are correct?

(a) i、ii、iv (b) i、iv、v (c) ii、iv、v

(d) i、ii、iii、v (e) i、v

2. Let $\mathrm{B} \in \mathcal{R}^{n\times n}$ be an orthogonal matrix :

[i] The eigenvalues of **B** are 1, 0, or -1

[ii] The determinant of **B** is 1

[iii] The columns of **B** form an orthonormal basis of $R^n$

[iv] For any $\mathrm{x} \in \mathcal{R}^{n\times 1}$, $\|\mathrm{x}\| = \|\mathrm{Bx}\|$

[v] For any $\mathrm{x}, \mathrm{y} \in \mathcal{R}^{n\times 1}$, $\langle \mathbf{x},\mathbf{y} \rangle = \langle \mathbf{Bx},\mathbf{By} \rangle$; where $\langle \mathbf{x},\mathbf{y} \rangle = \mathbf{y}^T\mathbf{x}$ is the inner product of vectors **x** and **y**.

Which statements are always correct?

(a) i、 iv、v (b) ii、iii、iv (c) iii、iv、v

(d) i、ii、v (e) i、ii、iii

3. In the following, which is NOT diagonalizable?

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

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4. Consider three vectors in $R^4$: $[1, 0, 1, 0]^T$, $[0, 1, -2, 1]^T$, $[1, -1, 0, 0]^T$. Among the following vectors, which form an orthonormal basis of the subspace spanned by three vectors?

   [i] $[\ 1/\sqrt{2},\ \ 0,\ \ 1/\sqrt{2},\ \ 0\ ]^T$
   
   [ii] $[\ 0,\ \ 1/\sqrt{6},\ \ -2/\sqrt{6},\ \ 1/\sqrt{6}\ ]^T$
   
   [iii] $[\ 1/\sqrt{6},\ \ -2/\sqrt{6},\ \ -1/\sqrt{6},\ \ 0\ ]^T$
   
   [iv] $[\ 1/\sqrt{2},\ \ -1/\sqrt{2},\ \ 0,\ \ 0\ ]^T$
   
   [v] $[\ 1/2,\ \ 1/2,\ \ -1/2,\ \ 1/2\ ]^T$

   (a) i、iii、v　(b) i、ii、v　(c) iii、iv、v
   
   (d) i、ii、iv　(e) i、iii、iv

5. Given $n \times n$ matrices **A** and **B**, and there is an $n \times n$ invertible matrix **P** such that $\mathbf{B} = \mathbf{P}^{-1}\mathbf{AP}$.

   [i] **A** and **B** have the same trace
   
   [ii] **A** and **B** have the same eigenvectors
   
   [iii] **A** and **B** have the same determinant
   
   [iv] **A** and **B** have the same eigenvalues
   
   [v] **A** and **B** are simultaneously diagonalizable

   Among the above statements, which are not always true?

   (a) ii、iv、v　(b) i、iv、v　(c) i、iii、iv
   
   (d) ii、v　(e) ii、iii

6. Let **A** be an $n \times n$ matrix with $n$ real eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_n > 0$,

   [i] For any nonzero $n \times 1$ vectors **x**, $\lambda_n \le \frac{\mathbf{x}^H \mathbf{A}\mathbf{x}}{\|\mathbf{x}\|^2} \le \lambda_1$
   
   [ii] The determinant of $\mu\mathbf{A}^2$ is $\mu\lambda_1^2\lambda_2^2\cdots\lambda_n^2$
   
   [iii] The eigenvalues of matrix $(\mathbf{A}+\mathbf{I})^{-1}$ are the same as those of $\mathbf{A}^{-1}$
   
   [iv] The eigenvectors of matrix $(\mathbf{A}+\mathbf{I})^{-1}$ are the same as those of $\mathbf{A}^{-1}$
   
   [v] Given a nonzero $n \times 1$ vector **v** and for any nonzero $n \times 1$ vectors **x**, $\frac{|\mathbf{v}^T\mathbf{x}|^2}{\mathbf{x}^T\mathbf{A}\mathbf{x}} \le \mathbf{v}^T\mathbf{A}^{-1}\mathbf{v}$

   Among above statements, which are not always true?

   (a) ii、iii　(b) iii、v　(c) iii、iv、v
   
   (d) iii、iv　(e) i、ii

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科目：線性代數【通訊所碩士班甲組】

計算證明題 (70%)

1. Given singular value decomposition of a matrix $\mathbf{H} \in \mathcal{C}^{m\times n}$ as $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are $m \times m$ and $n \times n$ unitary matrices, $\Sigma$ is an $m \times n$ diagonal matrix composed of nonnegative singular values $\sigma_1, \cdots, \sigma_r, 0, \cdots, 0$, where $r = \text{rank}(\mathbf{H})$.
   (a) Find the eigenvalues of $\mathbf{H}\mathbf{H}^H$. (5%)
   (b) Prove that (5%)
$$\sum_{i=1}^{r} \sigma_i^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} |h_{i,j}|^2 .$$

2. Find the LU decomposition of the following matrix (10%):
$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

3. Please use LU–decomposition to solve the following system of linear equations (10%):
$$\begin{cases} -X_1 + 2X_2 - X_3 = 2 \\ X_1 - 4X_2 + 6X_3 = -3 \\ -2X_1 + 6X_2 - 6X_3 = 8 \end{cases}$$

4. Let $B_0 = \{\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\}$, $B_1 = \{\begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}3\\-1\end{bmatrix}\}$ and $B_2 = \{\begin{bmatrix}2\\3\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix}\}$ be three bases in $R^2$.
   Let $X = \begin{bmatrix}5\\2\end{bmatrix}$ in $B_0$
   (a) Write X in term of the vector in $B_2$. (5%)
   (b) Find the transformation matrix that converts a vector from in terms of Base $B_1$ to Base $B_2$. (5%)

5. Consider the vector space $R^3$ with Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vector $u_1 = (0,1,0)$, $u_2 = (1,1,1)$ and $u_3 = (1,1,2)$ into an orthogonal basis $\{v_1,v_2,v_3\}$; then normalize the orthogonal basis vectors to obtain an orthonomal basis $\{q_1,q_2,q_3\}$ (10%).

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6. Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$. (10%)

7. Consider the vector space $P_3$ of polynomials of degree less than 3, and the ordered basis $B = \{x^2, x, 1\}$ for $P_3$. Let T: $P_3 \rightarrow P_3$ be the linear transformation such that

$$T(ax^2 + bx + c) = (a - c)x^2 - bx + 2c$$

Find the eigenvalues and the eigenvectors for the linear transformation T. (10%)

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科目：機率【通訊所碩士班甲組】

1. (10%) A pair of fair dice is rolled six times. What is the probability that "nine" will not show at all?

2. (10%) Let $X$ be a random variable with exponential distribution, derive and find the mean and variance of $6X$.

3. (10%) Let $X$ be a normal random variable with mean 10 and variance 9. How can we design a random variable $Y$ such that $Y$ is normal distributed with mean 100 and standard deviation 2.

4. (10%) Let $X$ and $Y$ be independent uniform random variables with ranges [0, 2] and [0, 6] respectively. Derive and plot the probability density function of the random variable $Z = X + Y$.

5. (10%) Let $X$ and $Y$ be independent normal random variables with mean 0 and variance 6. Derive and plot the probability density functions of the random variables $Z = \sqrt{X^2+Y^2}$ and $W = Y/X$

6. (10%) Given a real-valued random variable $X$ with finite second moment. Identify all the true statements:

   (a) $E\{X^2\} \le (E\{X\})^2$ ;
   (b) $E\{cX\} \ne cE\{X\}$, where $c$ is a constant value;
   (c) $E\{\log(1+X)\} \le \log(1+E\{X\})$.

7. (10%) Given a condition so that the fact $E\{\frac{1}{X}\} = \frac{1}{E\{X\}}$ is true.

8. (10%) Let $X$ and $Y$ be independent normal random variables with zero mean and unit variance. Find the value of $E\{X^2Y + XY^2\}$, in which $E\{\cdot\}$ takes the expectation with respect to $X$ and $Y$.

9. (10%) Given a real-valued random variable $X$ with finite second moment $E\{X^2\}$. Show the conditions on $c$ so that the following statement is true:

$$E\{X^2\} \le cE\{X^2\} \text{ if and only if } E\{X^2\} = 0.$$

10. (10%) Let $Y$ be a binomial distribution with parameters $n$ and $p$; i.e., the probability distribution function of $Y$ is given by $P(Y=y) = \binom{n}{y} p^y (1-p)^{n-y}$, $y = 0, 1, 2, \cdots, n$. Show the probability generating function of $Y$.

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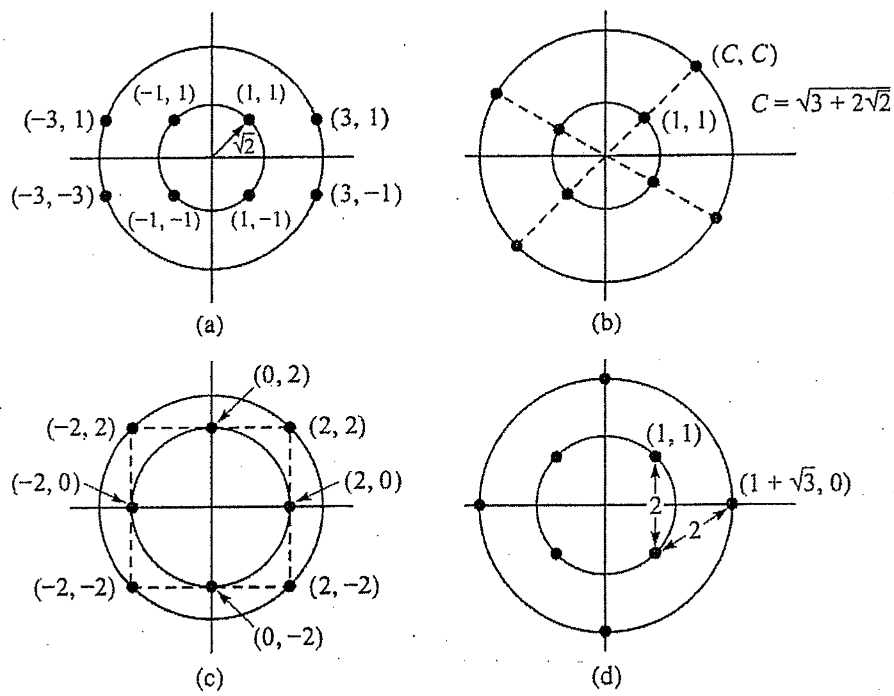
科目：通訊理論【通訊所碩士班甲組】

## 通訊理論 （Communications Theory）

1. [20] **Nyquist Pulse-Shaping Criterion:** Let the symbol duration $T = 1$. Which of the following signals are Nyquist pulses for zero-ISI? Please explain your answers.

(a)[4] $\frac{\sin \pi t}{\pi t}$ (b)[4] $\frac{\sin \pi t}{\pi t}\exp\{-2t^2\}$ (c)[4] $\sin \pi t + \cos \pi t$ (d)[4] $\exp\left\{-\frac{t^2}{2}\right\}$ (e)[4] $\left(\frac{\sin \pi t}{\pi t}\right)^4$

2. [20] **Signal Constellation:**
   (a) [10] Please design the Gray coding for an 8-PSK modulation scheme.
   (b) [10] Which of the following 8-QAM modulation schemes has the best performance? Please explain your answer. (Assume that all signal points are equally probable.)




3. [20] **Optimum Receivers for AWGN Channels:** Please derive the error probability for $M$-ary biorthogonal signaling that adopts optimal detection. All signals are equiprobable and have equal energy. The noises are i.i.d. Gaussian random variables with zero-mean and variance $\frac{N_0}{2}$. (You do not have to show a closed form expression.)

4. [20] **Explanations:**
   (a) [2] What is an energy signal?
   (b) [2] What is a power signal?
   (c) [2] What kind of system is said to be causal?
   (d) [2] What kind of system is said to be stable?
   (e) [2] What is an Ergodic process?
   (f) [2] What is a cyclostationary process?
   (g) [2] Please describe the condition that two events are statistically independent.
   (h) [2] Please describe the condition for two random variables $X$ and $Y$ to be orthogonal.
   (i) [2] Please describe the conditions for a random process to be wide-sense stationary (WSS).
   (j) [2] Please describe the Offset QPSK modulation scheme.

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科目：通訊理論【通訊所碩士班甲組】

5. [20] **Fourier Transform:** (Hint: You may use the attached properties of the Fourier transform.)
   (a) [10] Please show that the Fourier transform of a decaying exponential pulse is given by:
   $$\exp(-at)u(t) \rightleftharpoons \frac{1}{a+j2\pi f},\ a>0, \text{ where } u(t)=\begin{cases}1, & t>0\\ \frac{1}{2}, & t=0.\\ 0, & t<0\end{cases}$$
   (b) [10] Please show that the Fourier transform of a double exponential pulse is given by:
   $$\exp(-a|t|) \rightleftharpoons \frac{2a}{a^2+(2\pi f)^2},\ a>0.$$

## Properties of the Fourier Transform

| Property | Mathematical Description |
|---|---|
| Linearity | $ag_1(t)+bg_2(t) \rightleftharpoons aG_1(f)+bG_2(f)$, where $a$ and $b$ are constants. |
| Time scaling | $g(at) \rightleftharpoons \frac{1}{\lvert a\rvert}G\left(\frac{f}{a}\right)$, where $a$ is a constant. |
| Duality | If $g(t) \rightleftharpoons G(f)$, then $G(t) \rightleftharpoons g(-f)$. |
| Time shifting | $g(t-t_0) \rightleftharpoons G(f)\exp(-j2\pi f t_0)$. |
| Frequency shifting | $\exp(j2\pi f_c t)g(t) \rightleftharpoons G(f-f_c)$. |
| Area under $g(t)$ | $\int_{-\infty}^{\infty} g(t)\,dt = G(0)$. |
| Differentiation in the time domain | $\frac{d}{dt}g(t) \rightleftharpoons j2\pi f G(f)$. |
| Integration in the time domain | $\int_{-\infty}^{t} g(\tau)\,d\tau \rightleftharpoons \frac{1}{j2\pi f}G(f)+\frac{G(0)}{2}\delta(f)$. |
| Conjugate functions | If $g(t) \rightleftharpoons G(f)$, then $g^*(t) \rightleftharpoons G^*(-f)$. |
| Multiplication in the time domain | $g_1(t)g_2(t) \rightleftharpoons \int_{-\infty}^{\infty} G_1(\lambda)G(f-\lambda)\,d\lambda$. |
| Convolution in the time domain | $\int_{-\infty}^{\infty} g_1(\tau)g_2(t-\tau)\,d\tau \rightleftharpoons G_1(f)G_2(f)$. |
| Rayleigh's energy theorem | $\int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(f)\rvert^2 df$. |

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科目：工程數學甲【通訊所碩士班乙組】

**Problem 1 Multiple Choice (30%)**

Instructions:

- There are 10 questions, each of which is associated with 4 possible responses.
- For each of questions, select **ONE most appropriate** response.
- For each response you provide, **you will be awarded 3 marks if the response is correct and -3 marks if the response is incorrect (答錯一題倒扣三分).**
- You get 0 mark if no response is provided.

(1.1) What is the solution of the ODE $\frac{dy}{dx} = 2xy^2$ for which y(0)=1?

(A) $y(x) = (1+x)^{-1}$. (B) $y(x) = (1-x^2)^{-1}$. (C) $y(x) = e^{-x}$. (D) $y(x) = 1 + \sqrt[3]{x^4}$.

(1.2) What is the amplitude of the sinusoidal solution of $\frac{dx}{dt} + 2x = 5\sin(3t)$ ?

(A) $\frac{2}{5}$ (B) 1 (C) $\sqrt{7}$ (D) $\frac{5}{\sqrt{13}}$

(1.3) Let $\boldsymbol{L}[\cdot]$ denotes the Laplace transform.

(A) The Laplace transform is a linear operation.

(B) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}[f(t-1)] = F(s-1)$.

(C) $\boldsymbol{L}[(t-2)] = e^{-s}/s$.

(D) None of the above statements is FALSE.

(1.4) What is the general solution to the ODE $t\frac{dx}{dt} = 4t - 3x$ ?

(A) x(t) = constant. (B) $x(t) = -ct^3$. (C) $x(t) = ct^{-3} + t$. (D) $x(t) = -ce^{3t} + 4t$.

(1.5) Let $\boldsymbol{L}[\cdot]$ denotes the Laplace transform.

(A) $\boldsymbol{L}[t^{-0.5}] = \sqrt{\frac{\pi}{s}}$.

(B) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}\left[f\left(\frac{t}{a}\right)\right] = F(as)$.

(C) If $\boldsymbol{L}[f(t)] = F(s)$, then $\boldsymbol{L}\left[\frac{df}{dt}(t)\right] = sF(s)$.

(D) All of the above statements are TRUE.

(1.6) Define the *del operator* $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$, and consider $\varphi(x,y,z) = (\cos z^3)e^{\sqrt{x+y}}$.

(A) $\nabla \cdot \nabla\varphi = \varphi$. (B) $\nabla \times \nabla\varphi = 0$. (C) $\mathbf{R} \cdot \nabla\varphi = 0$, where $\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(D) None of the above statements is TRUE.

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(1.7) Consider the del operator defined in (1.6).

(A) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$, where $\mathbf{F}$ is a vector field with continuous first and second derivatives.

(B) $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$, where $\mathbf{F}$ and $\mathbf{G}$ are two smooth vector fields.

(C) $\nabla \cdot (\varphi \mathbf{F}) = \nabla\varphi \cdot \mathbf{F} + \varphi(\nabla \cdot \mathbf{F})$, where $\varphi$ and $\mathbf{F}$ are smooth scalar and vector fields, respectively.

(D) All of the above statements are TRUE.

(1.8) Consider the ODE $\ddot{x} + x^2\dot{x} + x(x^2 - 1) = 0$.

(A) This is a linear ODE.

(B) This is a time-varying ODE.

(C) This ODE has three equilibria.

(D) The equilibria of this ODE are $\pm 1$ and $\pm 2$.

(1.9) Consider the heat equation

$$\frac{\partial u}{\partial t}(x,t) = 4\frac{\partial^2 u}{\partial x^2}(x,t) \qquad \forall 0 < x < 1, t > 0 \qquad (1.9.1)$$

$$u(0,t) = u(1,t) = 0 \qquad \forall t > 0 \qquad (1.9.2)$$

$$u(x,0) = f(x) \qquad \forall 0 < x < 1 \qquad (1.9.3)$$

(A) Without considering the boundary condition (1.9.3), a general solution to the heat equation is $u(x,t) = \sum_{n=1}^{\infty} C_n \sin(n\pi x)\, e^{-4n^2\pi^2 t}$.

(B) Suppose $f(x) = 7\sin(3\pi x)$. Then $u(x,t) = 7\sin(3\pi x)\, e^{-4n^2\pi^2 t}$.

(C) Both of the above statements are TRUE.

(D) None of the above statement is TRUE.

(1.10) Consider the wave equation

$$\frac{\partial^2 w}{\partial t^2}(x,t) = 3\frac{\partial^2 w}{\partial x^2}(x,t) \qquad \forall 0 < x < 1, t > 0 \qquad (1.10.1)$$

$$w(0,t) = w(1,t) = 0 \qquad \forall t > 0 \qquad (1.10.2)$$

$$w(x,0) = f(x) \qquad \forall 0 < x < 1 \qquad (1.10.3)$$

$$\frac{\partial w}{\partial t}(x,0) = 0 \qquad \forall 0 < x < 1 \qquad (1.10.4)$$

(A) Without considering the boundary condition (1.10.3), a general solution to the wave equation is $w(x,t) = \sum_{n=1}^{\infty} C_n \sin(n\pi x)\cos(n\pi t)$.

(B) Suppose $f(x) = 17\sin(9\pi x)$. Then $w(x,t) = 17\sin(9\pi x)\,(\sin(27\pi t) + 1)$.

(C) Both of the above statements are TRUE.

(D) None of the above statement is TRUE.

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科目：工程數學甲【通訊所碩士班乙組】

**Problem 2 (15%)**

Consider the vector function $\mathbf{F}(x,y) = (y^3 - 12y)\mathbf{i} + (15x - x^3)\mathbf{j}$.

**(2.1) (10%)** Find the simple closed curve C for which the integral

$$\oint_C \mathbf{F} \bullet d\mathbf{r}$$

(with positive orientation) will have the largest positive value. (Hint: Use Green's Theorem)

**(2.2) (5%)** Compute this largest positive value.

**Problem 3 (13%)**

This problem has two sub-problems. **Please give your answers in details.**

(a) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in \mathbb{R}^{2\times 2}$ and define a scalar-valued function $f_A(\mathbf{x},\mathbf{y}) := \mathbf{x}^T A\mathbf{y}$ for any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$. If $f_A(\mathbf{x},\mathbf{y})$ is an inner product on $\mathbb{R}^2$, then what conclusions can be made on all the entries $a_{ij}$? (6%)

(b) Consider the inner product space $\left(\mathbb{R}^{2\times 2}, \langle\cdot,\cdot\rangle\right)$ with $\langle A,B\rangle := trace(AB^T)$ defined for matrices $A$ and $B$ in $\mathbb{R}^{2\times 2}$. Let $S$ be the subspace of $\left(\mathbb{R}^{2\times 2}, \langle\cdot,\cdot\rangle\right)$ defined as $S := \left\{A \in \mathbb{R}^{2\times 2} \,\middle|\, A = A^T\right\}$. Let $E$ denote the standard basis for $S$ and $F$ denote an orthonormal basis for $S$ that is derived from basis $E$. What are bases $E$ and $F$? (7%)

**Problem 4 (12%)**

Given real numbers $a_i, b_i, c_i$ for $i = 1, 2$, let $L$ be a transformation from $V$ to $W$, with $V = \mathbb{R}^2 = W$, defined by

$$L(\mathbf{r}) := \begin{bmatrix} a_1 r_1 + b_1 r_2 + c_1 \\ a_2 r_1 + b_2 r_2 + c_2 \end{bmatrix}, \quad \forall\, \mathbf{r} := \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \in \mathbb{R}^2.$$

Suppose that $c_1$ and $c_2$ are of the values so that $L: V \to W$ is linear.

**以下小題僅需依序寫下答案即可，不需做任何推導。**

(a) Let $E$ be the standard basis for $\mathbb{R}^2$. Find the matrix $A$ representing $L$ with respect to basis $E$ for both $V$ and $W$. (5%)

(b) Let $F := \{\mathbf{f}_1, \mathbf{f}_2\}$ be another basis for $\mathbb{R}^2$ and let $Q$ denote the matrix representation of $L$ with respect to basis $F$ for $V$ and basis $E$ for $W$, respectively. Denote $P := \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$. Please write the algebraic relationship between $Q$, $P$, $\mathbf{f}_1$, and $\mathbf{f}_2$. (7%)

**Problem 5 (15%)**

Compute the least upper bound of the integral

$$\left| \int_C (e^z - \bar{z})\, dz \right|$$

where $z$ is a complex variable, $\bar{z}$ is its complex conjugate, and $C$ denotes the boundary of a triangle with vertices at the points $i3$, $-4$ and 0, oriented in counterclockwise direction.

**Problem 6 (15%)**

Let $F(\omega)$ be the Fourier transform of $f(t)$. Compute $\mathscr{F}\left(i\cdot t\cdot f(t)\right)$, where $\mathscr{F}$ stands for the Fourier transform and $i = \sqrt{-1}$. Write down you answer in terms of $F(\omega)$, and each calculation step is also required.

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科目：電子學【通訊所碩士班乙組】

1. (25%) Consider a source follower such as that in Fig. 1 Specifically,
   (a) Please derive $R_{in}$, $A_{vo}$, $A_v$, and $R_o$ with $r_o$ taken into account. (4*5%)
   (b) Please derive the overall small-signal voltage gain $G_v$ with $r_o$ taken into account. (1*5%)

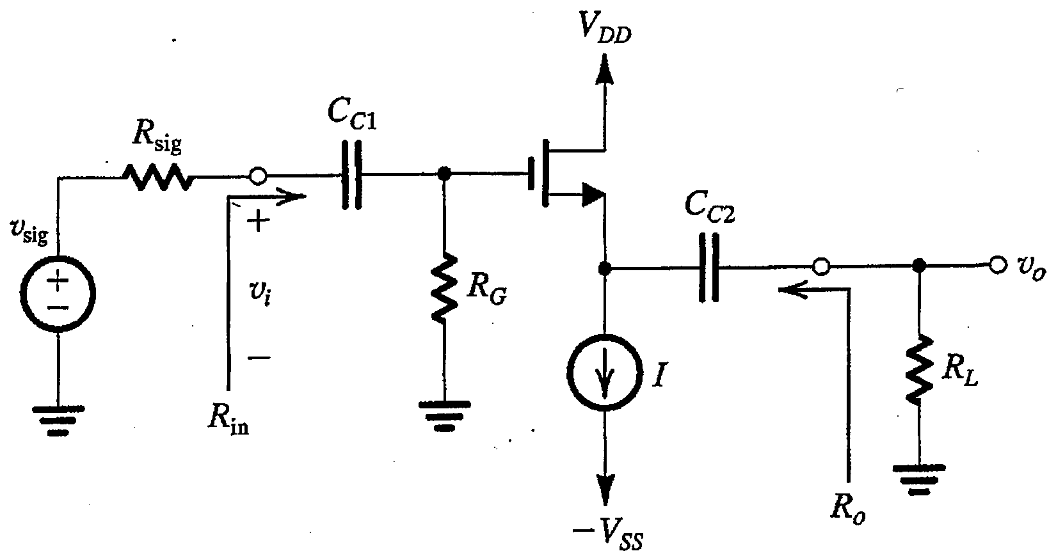


**Fig. 1**

2. (20%) Please find the three break frequencies and the low *3dB* frequency of the common emitter amplifier with an emitter resistance as shown in Fig. 2. (4*5%)

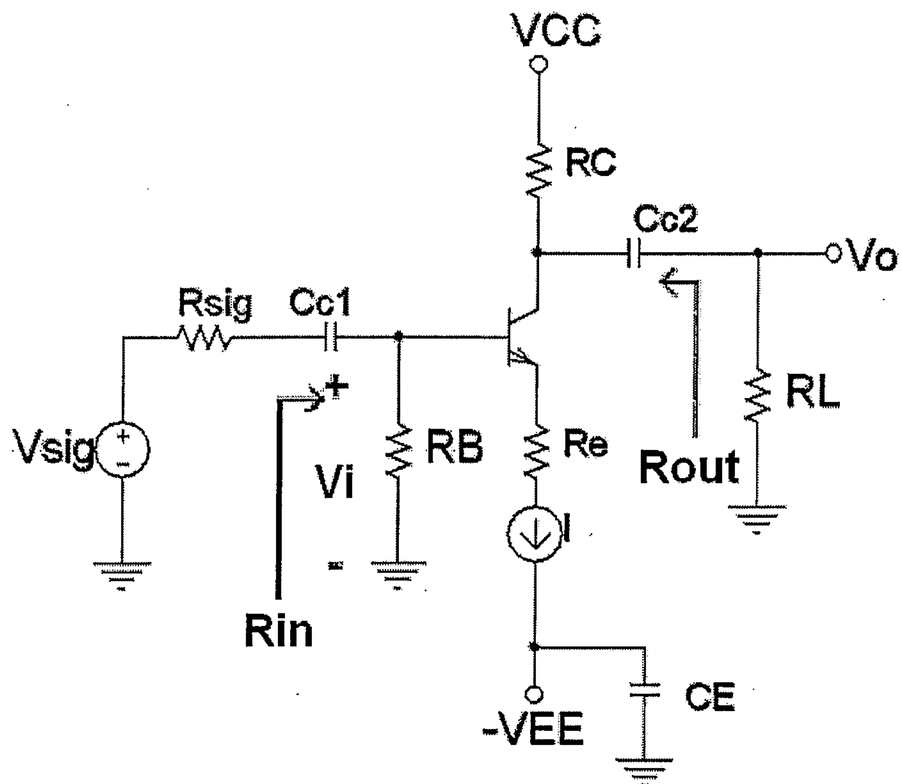


**Fig. 2**

# 國立中山大學100學年度碩士班招生考試試題

科目：電子學【通訊所碩士班乙組】

3. (20%) Design and draw a three input CMOS NAND and CMOS NOR gates respectively. Also, please find out the $(W/L)_P/(W/L)_N$ ratios of the designed CMOS NAND and CMOS NOR gates respectively so that their PMOS part and NMOS part have same MOS transconductance parameters, where $W/L$ is aspect ratio. (2*5%, 1*10%)

4. (15%) In Fig. 4, given that the input resistance is 10 KΩ and the differential voltage gain is 100, please find out (a) $R_1 = R_3 = ?$, and (b) $R_2 = R_4 = ?$ (2*7.5%)

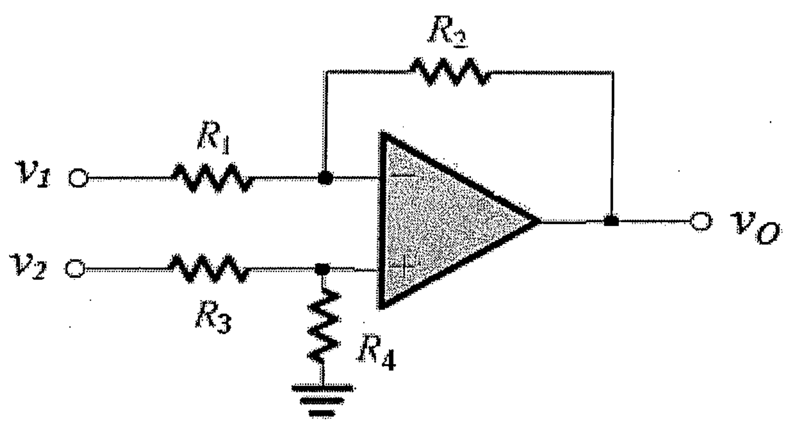


**Fig. 4**

5. (20%) Fig. 5 shows an operation amplifier and its small equivalent circuit. Please derive its output resistance $R_o$ and its dc open-loop gain $A_v$ with respect to the circuit transistor's $g_m$ and $r_o$. (2*10%)

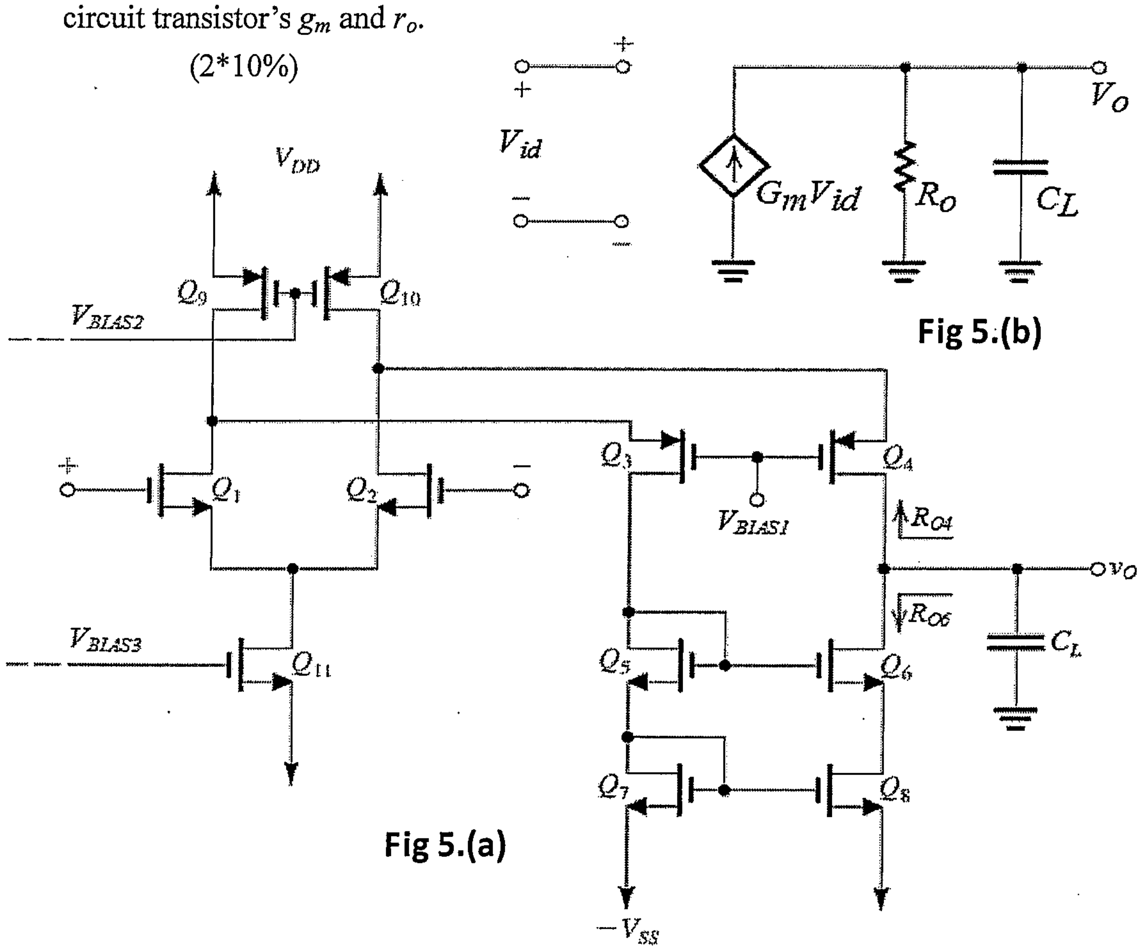


**Fig 5.(b)**



**Fig 5.(a)**

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1. Please answer questions about static electric fields.
   (a) (5%) Under what conditions will the electric field intensity be both solenoidal and irrotational?
   (b) (5%) If the electric potential at a point is zero, dose it follow that the electrical field intensity is also zero at that point? Explain.
   (c) (5%) Why are there no free charges in the interior of a good conductor under static conditions?
   (d) (5%) If $\nabla^2 \mathbf{U} = 0$, why does it not follow that **U** is identically zero?
   (e) (5%) Assume that fixed charges +Q and –Q are deposited on the plates of an isolated parallel-plate capacitor.
   (i) Dose the electric field intensity in the space between the plates depend on the permittivity of the medium?
   (ii) Dose the electric flux density depend on the permittivity of the medium? Explain.

2. Please answer questions about static magnetic fields.
   (a) (5%) Which postulate of magnetostatics denies the existence of isolated magnetic charges?
   (b) (5%) State the law of conservation of magnetic flux. (5%)
   (c) (5%) Does the magnetic field intensity due to a current distribution depend on the properties of the medium?
   (d) (5%) What is meant by the internal inductance of a conductor?
   (e) (5%) What is the relation between the force and the stored magnetic energy in a system of current-carrying circuits under the condition of constant flux linkages?

3. To investigate the electromagnetic coupling of cellular phone antennas and a human head, a phantom head – a plastic container filled with a solution that approximately resembles the dielectric and conductive properties of a human head – is used for measurements. In particular, solutions are made that have the relative permittivity and loss tangent equal to the corresponding average head tissue parameters at two frequency bands allocated for wireless communications in North America: (i) $\varepsilon_r = 44.8$ and $\tan\delta_c = 0.408$ at $f = 835$ MHz and (ii) $\varepsilon_r = 41.9$ and $\tan\delta_c = 0.293$ at $f = 1.9$ GHz. Assume that the phantom solution has the same permeability as that of a vacuum.
   (a) (5%) Find the attenuation constant of a uniform plane wave propagating through the phantom solution.
   (b) (10%) If the rms electric field intensity of the wave at its entry into the solution is $E_0 = 50$ V/m, determine the time-average power absorbed in the first cm of depth into the solution per 1 $\text{cm}^2$ of cross-sectional area, that is, in the first 1 cm × 1 cm × 1 cm of the solution past the interface, at each of the frequencies.

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4. To determine the frequency, $f$, and electric-field rms intensity, $E_{i0}$, of a uniform plane wave traveling in air, a perfectly conducting plate is introduced normally to the wave propagation and electromotive force (emf) induced in a small square wire loop of area 6.25 $cm^2$ is measured. By varying the orientation and location of the loop, it is found that the rms emf in it has a maximum of 5 mV at a distance of 80 cm from the conducting plate (with the plane of the loop being perpendicular to the magnetic field vector of the wave). It is also found that the first adjacent minimum (zero) of the rms emf is at 60 cm from the conducting plate (for the same orientation of the loop).
   (10%) What are $f$ and $E_{i0}$?

5. RG-402U semi-rigid coaxial cable has an inner conductor of 0.91 mm, and a dielectric diameter (equal to the inner diameter of the outer conductor) of 3.02 mm. Both conductors are copper with conductivity of $5.8\times10^7$ S/m, and the dielectric material is Teflon with dielectric constant of 2.08 and loss tangent of 0.0004.
   (10%) Find the characteristic impedance (in ohm) and the attenuation (in dB/m) of the line at 1 GHz.

6. Consider a $TE_{02}$ mode propagating through an air-filled rectangular metallic waveguide of transverse dimensions $a = 38.1$ cm and $b = 190.05$ cm (WR-1500 waveguide).
   (a) (5%) Determine the cutoff frequency of this mode.
   (b) (10%) Find the power-handling capacity of the waveguide for this mode, i.e., the maximum time-average power that can be carried by the $TE_{02}$ mode prior to an eventual dielectric breakdown at a frequency of $f = 1.8$ GHz. Note that the dielectric strength of air is $E_{cr0} = 3\times10^6$ V/m).