國立中山大學九十一學年度碩士班招生考試試題

科目：工程數學（含線性代數及機率）（通訊所甲組）共 1 頁 第 1 頁

國立中山大學九十一學年度碩博士班招生考試試題

科目: 工程數學(通訊所碩士班) 甲組

1. (Totally 10 points) To get full credits, you have to answer the questions in terms of mathematics explicitly and accurately.
   (a) (5 points) What is the weak law of large numbers in probability theory?
   (b) (5 points) What is the central limit theorem in probability theory?

2. (Totally 20 points) Let X and Y be two independent Gaussian random variables, where E[X]=E[Y]=0 and Var(X)=Var(Y)=1. Denote the probability that X is smaller than x by Pr(X<x). Let Δx and Δy be two sufficiently small positive real numbers.
   (a) (5 points) Prove that $\Pr(x \leqq X \leqq x+\Delta x,\ y \leqq Y \leqq y+\Delta y)=(1/(2\pi))\exp(-0.5(x*x+y*y))\Delta x\Delta y$
   (b) (15 points) Let Z=X+iY, where i is the square root of −1. Calculate $\Pr(|Z| \leqq 1)$

3. (Totally 15 points)
   (a) (5 points) What is a basis of a vector space?
   (b) (10 points) Consider the regular two-dimensional Euclidian space. Let v(1)=[3 4] and v(2)=[1 1]. First, normalize v(1) and then use Gram-Schmidt procedure to derive an orthonormal basis from {v(1),v(2)}

4. (Totally 55 points) Let X(1), X(2), …,X(n), X(n+1) … be an infinite sequence of random variables having a common state space {0,1}. Namely, in any realization, X(i) is either zero or one. In addition, X(1), X(2), …,X(n), X(n+1) … form a time-homogeneous Markov Chain in the following sense. For any n greater than one and any real numbers x(n+1), x(n),..,x(2),x(1), Pr(X(n+1)=x(n+1)|X(n)=x(n),X(n-1)=x(n-1),…,X(1)=x(1))=Pr(X(n+1)=x(n+1)|X(n)= x(n))=Pr(X(2)=x(n+1)|X(1)=x(n)). Furthermore, Pr(X(2)=1|X(1)=0)=1/3 and Pr(X(2)=0|X(1)=1)=1/4. Let A be a 2 by 2 matrix such that the element in the intersection of the (p+1)-th row and (q+1)-th column is Pr(X(2)=q|X(1)=p).
   (a) (5 points) Calculate the eigenvalues of the matrix A
   (b) (10 points) Calculate two linearly independent eigenvectors with unit norm
   (c) (10 points) Suppose that Pr(X(1)=0)=1/2. Calculate Pr(X(3)=0).
   (d) (10 points) Suppose that when n is large enough, Pr(X(n+1)=0)=Pr(X(n)=0)=u(0), where u(0) is a positive real number. Calculate u(0). (Hint: Use conditional probability)
   (e) (20 points) Let A^{n}=A*A*A…*A, in which there are n identical matrices multiplied in the right hand side. Suppose that when n goes to infinity, A^{n} converges to a 2 by 2 matrix B. Calculate B. (Hint: Use diagonization)

国立中山大學九十一學年度碩士班招生考試試題

科目：通訊系統（通訊所）（甲組）

1. (20%) Compare Amplitude modulation, SSB modulation and DSB modulation in terms of (a) Bandwidth (b) Transmitter complexity (c) Receiver complexity (d) Quality.

2. (20%) Consider a digital communication system in an AWGN channel. The signal-to-noise ratio is normally defined as E_b/N_o, which is assumed to be 10 dB at the receiver, where E_b is the energy per bit and N_o is the two-sided power spectral density of the noise. If the variance of the noise and bit duration both are unit, calculate actual signal level in volt received at the receiver.

3. (20%) The terms of 2ASK, 2PSK and 2FSK stand for three binary digital modulation schemes, which represent binary amplitude shift keying, binary phase shift keying and binary frequency shift keying, respectively. (i) Draw the schematic diagrams of the transmitters and receivers for the three schemes; (ii) Compare the three schemes in terms of (a) complexity; (b) BER performance in AWGN channel; (c) bandwidth required; (d) the capability against interference.

4. (20%) Answer the following questions in a concise yet accurate way:

   (a) Describe the differences between a baseband chip and a RF chip.

   (b) Should a modulation unit be implemented by using baseband chips or/and RF chips?

   (c) What is software radio? Give a simple description.

   (d) Write down the mathematical expressions for phase modulation (PM) and frequency modulation (FM).

   (e) What is the relation between the PM and FM?

5. (20%) Consider a two-path multipath channel in a communication system, where the relative delay between the two paths is one unit of time and the two paths have the same strength. (a) Draw the system block diagram of the channel; (b) Write down the impulse response of the channel; (c) Write down the transfer function of the two-path channel; (d) Explain the meaning of the term "frequency-selective fading" from the obtained results in (c).

科目：電子學（通訊所甲，乙組）

1. (a) Refer to Fig. 1, explain the function of the offset nulling terminals and sketch the offset nulling circuit. (3%)
   (b) Sketch the transfer characteristic v_O versus v_I for the circuits shown in Fig. 2. All diodes begin conducting at a forward voltage drop of 0.5 volt and have voltage drops of 0.7 volt when fully conducting. (6%)
   (c) What is Early effect? (3%)
   (d) An NMOS amplifier with depletion load and its transfer characteristic are depicted in Fig. 3. Indicate the operation modes of Q_1 and Q_2 in the regions I~IV. (3%)
   (e) The magnitude response of a capacitively coupled amplifier is shown in Fig. 4. Explain the reason for the magnitude decreasing at low and high frequencies. (3%)
   (f) Sketch the safe operation area of a BJT and explain it briefly. (3%)
   (g) The 741 op amp circuit is shown in Fig. 5. Indicate the short-circuit protection circuitry and explain its operations. (3%)
   (h) Give the transfer function of a second-order bandpass filter with a center frequency of 10^5 rad/s, a center-frequency gain of 10, and a 3-dB bandwidth of 10^3 rad/s. (3%)
   (i) Sketch a CMOS realization for the function $Y = \overline{A + B(C + D)}$. (3%)
2. A simple op amp circuit and its dc bias currents are shown in Fig. 6. Assume all the transistors have β=100, determine
   (a) the input resistance R_{id}(3%)
   (b) the voltage gain v_o/v_{id} (10%)
   (c) the output resistance R_O. (3%)
3. For the circuit in Fig. 7, $|V_t|$ =1V, $k'W/L$ =1mA/V^2, β =100, and the Early voltage for all devices (including those that implement the current sources) is 100V. The signal source V_s has a zero dc component.
   (a) Identify the feedback topology. (4%)
   (b) Find the values of open-loop gain A, feedback factor β, close-loop gain A_f, input resistance R_{in}, and output resistance R_{out}. (20%)
4. Consider the oscillator circuit in Fig. 8 and assume for simplicity that $\beta = \infty$. Find the frequency of oscillation and the minimum value of R_C (in terms of the bias current I) for oscillations to start. (10%)
5. In the TTL circuit shown in Fig. 9, the transistor parameters are β_F =20, β_R =0.1 (for each input emitter), $V\gamma$ =0.7V, V_{BE}(on) =0.7V, V_{BE}(sat) =0.8V, and V_{CE}(sat) =0.1V.
   (a) Calculate the maximum fanout for v_X=v_Y=5V. (10%)
   (b) Calculate the maximum fanout for v_X=v_Y=0.1V. (10%)
   (Assume v_O is allowed to decrease by 0.1V from the no-load condition.)

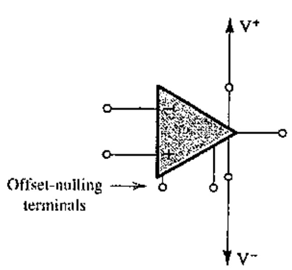

Figure 1

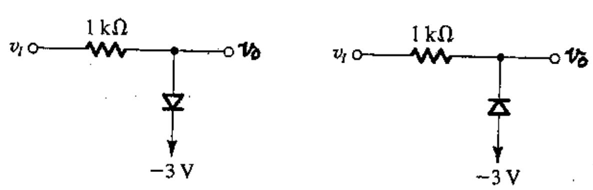

Figure 2

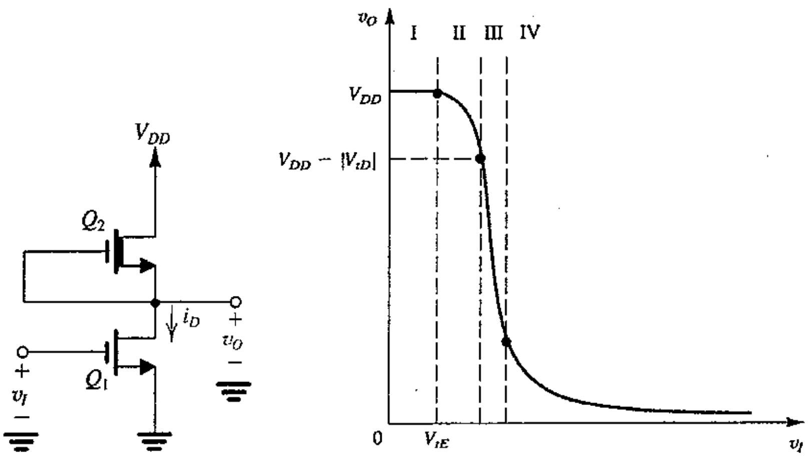

Figure 3

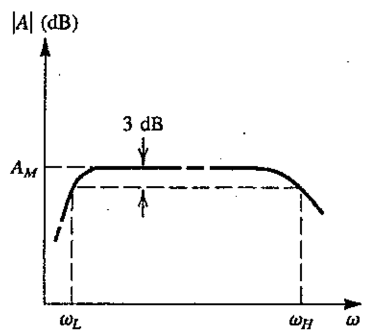

Figure 4

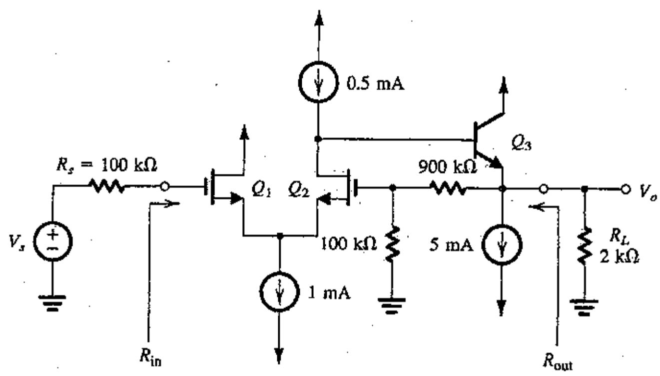

Figure 7

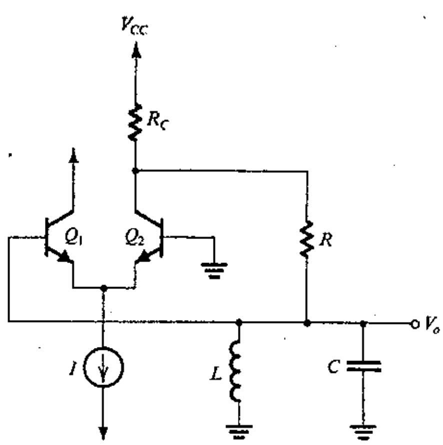

Figure 8

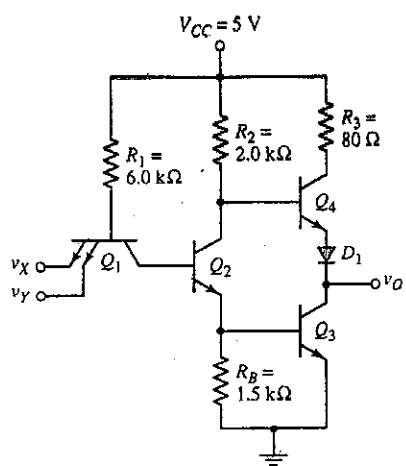

Figure 9

(Figures 5 and 6 are shown in next page)

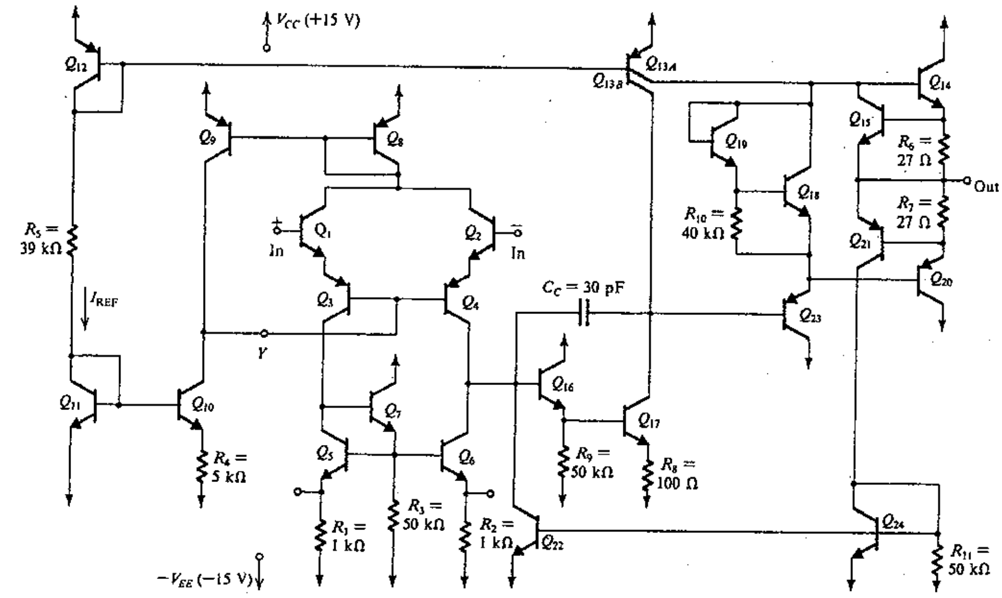

Figure 5

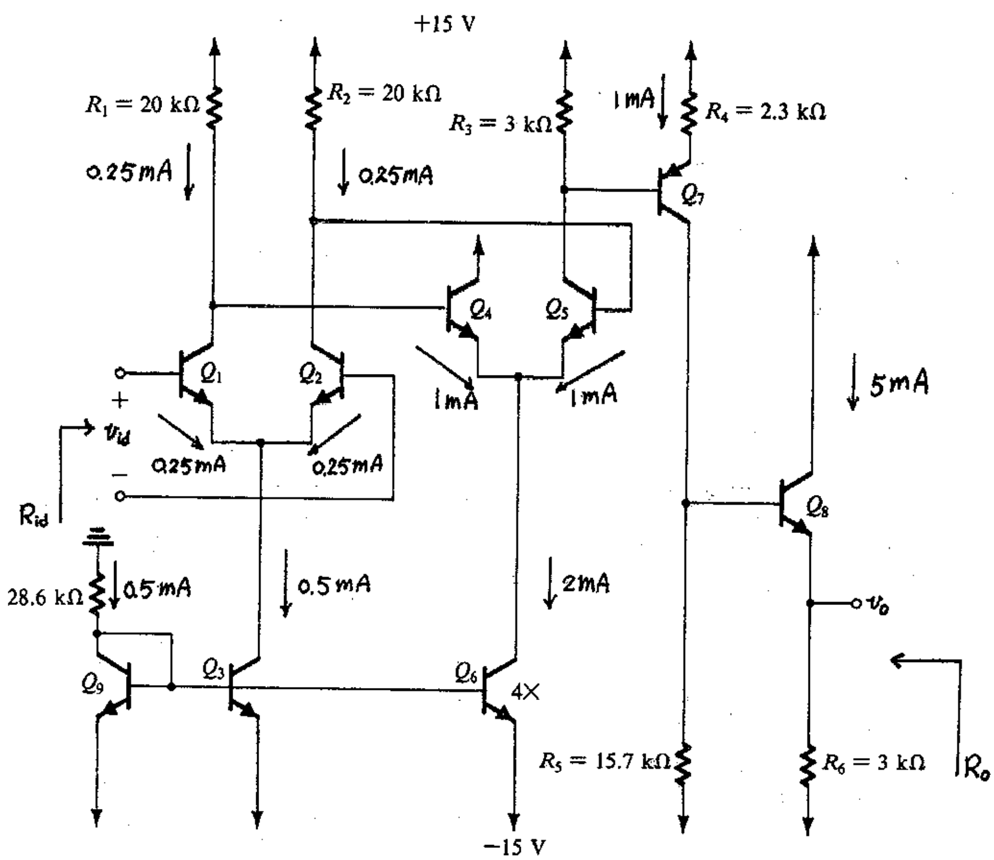

Figure 6

微分方程及向量分析

1. (15pts) For the differential equation
   $x^2 - 2x + xy'(x) + 2y(x) = 0$, obtain
   (a) the homogeneous solution, and (8pts)
   (b) the particular solutions. (7pts)
2. (20pts) Solve the following initial value problem
   $y''(t) + by'(t) + 10y(t) = 0$ with $y'(0) = 1$ and $y(0) = 0$, given
   (a) $b = 7$ (10pts)
   (b) $b = 6$ (10pts)
3. (20pts) We wish to solve the Laplace's equation using separation of variables
   $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ within $0 \le x \le b$ and $0 \le y \le a$ with boundary values given by
   $u(x=0, 0 \le y \le a) = u(x=b, 0 \le y \le a) = u(0 \le x \le b, y=0) = 0$ and
   $u(0 \le x \le b, y=a) = 1$.
   Let $u(x,y) = X(x)Y(y)$.
   (a) Show that $X''/X = \lambda$, where λ is a constant. (5pts)
   (b) Derive the boundary conditions for $X(0)$ and $X(b)$. (5pts).
   (c) Discuss whether $\lambda > 0$, $\lambda = 0$, or $\lambda < 0$. (5 pts)
   (d) Impose the boundary conditions for X to determine the possible values of λ. (5pts)
4. (15pts) Let $\vec{A}$ and f be vector and scalar fields, respectively.

   (a) Prove $\nabla \cdot \nabla \times \vec{A} = 0$. (5pts)

   (b) Express $\nabla \cdot (f\vec{A})$ in terms of f, $\vec{A}$, ∇f and $\nabla \cdot \vec{A}$. (5pts)

   (c) Suppose $\vec{A} = \vec{a}_x 2xy + \vec{a}_y x^2 + \vec{a}_z z$. Find an f such that $\vec{A} = \nabla f$. (5pts)
5. (15pts) We would like to evaluate the directional derivative of a scalar field
   $V(x,y,z) = xy + x + z + 1$ at the **origin.**
   (a) At first let us find the derivative along the direction $(1,1,1)$. Choose a nearby point $\Delta\vec{r} = (\Delta t, \Delta t, \Delta t)$.
   (i) Find ΔV, the increment of V from the origin to $\Delta\vec{r}$.
   (ii) Determine the derivative $\lim_{|\Delta\vec{r}| \to 0} \frac{\Delta V}{|\Delta\vec{r}|}$
   (b) Consider the derivative along any direction in the xy-plane. We should now use $\Delta\vec{r} = (\Delta x, \Delta y, 0)$. Which direction will give the maximum derivative?

6. (15pts) Verify divergence theorem for the vector field $\vec{F}(x,y,z)=\vec{a}_x x+\vec{a}_y 2x+\vec{a}_z xyz$ over a cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ and $z=1$.

(a) Compute $\nabla\cdot\vec{F}(x,y,z)$. (5pts)

(b) Perform the volume integral. (5pts)

(c) Perform the surface integral. (5pts)

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科目：電磁學（通訊所）（乙組）

1.(10%) Prove that the Lorentz condition for potentials is consistent with the equation of continuity.

2. A parallel-polarized wave is incident from air onto a dielectric with $\epsilon_r = 2.56$.

(a)(10%) Find the standing wave ratio for a normal incidence.

(b)(10%) Find the Brewster angle for the complete transmission.

(c)(10%) Calculate the reflection coefficient for a perpendicularly-polarized wave incident from air onto the same dielectric at the Brewster angle.

3. Consider a very long coaxial cable. The inner conductor has a radius a and is maintained at a potential V_0. The outer conductor has an inner radius b and is grounded. The conductors are assumed to have a surface resistivity R_s, and the material filling the space between the conductors is assumed to have a complex permittivity $\epsilon = \epsilon' - j\epsilon''$ and a permeability μ.

(a)(10%) Determine the potential distribution in the space between the conductors.

(b)(10%) Determine the electromagnetic fields of a traveling TEM wave inside the coaxial cable.

(c)(10%) Determine the transmission-line distributed parameters L, C, R and G.

4. Consider a length of air-filled copper X-band rectangular waveguide, with dimensions $a = 2.286$ cm, $b = 1.016$ cm. The conductivity of copper is $\sigma = 5.8 \times 10^7$ S/m.

(a)(10%) Find the cut off frequencies of the first four propagating modes.

(b)(10%) What is the attenuation in dB of a 1 m length of this guide when operating at $f = 10$ GHz?

(c)(10%) Determine the energy-transport velocity of the guide when operating at $f = 10$ GHz.