國立中山大學九十三學年度碩士班招生考試試題

科目：電子學　（通訊工程研究所碩士班甲、乙組）　

1. For the circuit shown in Fig. 1, find I_E and V_{CE} for $V_{BE} = 0.7$ V and (a) $\beta = \infty$ (b) $\beta = 10$. (20%)

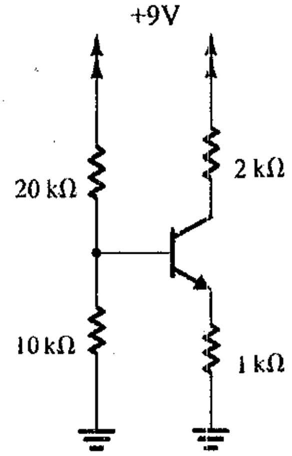

Figure 1

2. Consider the CMOS common-gate amplifier as shown in Fig. 2, using transistors for which $|V_t| = 1$ V, $\mu_n C_{ox} = 2\mu_p C_{ox} = 20\ \mu A/V^2$, $|V_A| = 50$ V, $L = 10\ \mu m$, $W_n = W_p = 100\ \mu m$ and $I_{REF} = 50\ \mu A$. For the input signal source having an average voltage of 0 V, what must V_{BIAS} be? For χ found to be 0.2, what are the values of the voltage gain v_o/v_i and the input resistance R_i? (20%)

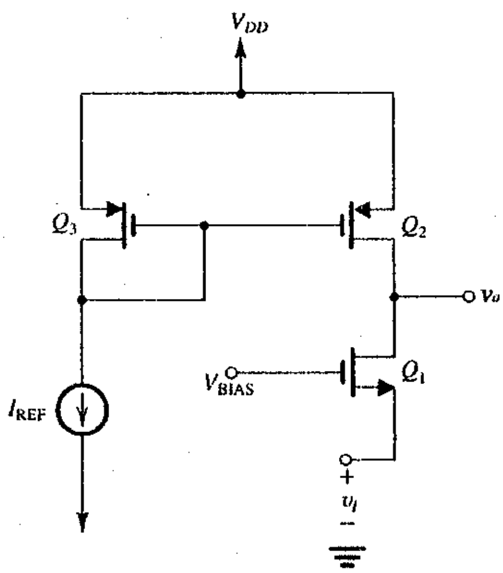

Figure 2

3. Sketch the circuit of a NAND SR flip-flop using CMOS, and prepare a truth table whose entries are in terms of stable output voltages available with a 3 V supply and devices for which $|V_t| < 3/2$ V. (20%)

4. Design an op-amp circuit with a gain of -2 V/V using three 100 kΩ resistors. How many solutions are there? Sketch all the solutions. What is the input resistance of each? (20%)

5. A manufacturing-process deviation in the production of TTL gates using the circuit of Fig. 3, reduces current gain such that $\beta_F = 9$ and $\beta_R = 0.05$. For input high, estimate I_{E2} for $V_{BE} = 0.7$ V and a load of 1 kΩ connected to the 5 V supply. What is the largest possible fanout (excluding the 1 kΩ load), for which saturation of Q_3 is still possible? (20%)

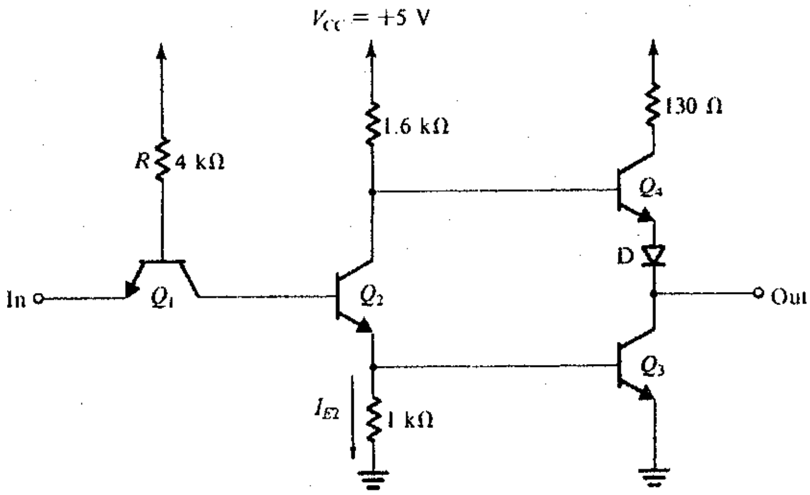

Figure 3

Problem 1. (Totally, 20 points)

Let V and W be two finite-dimensional vector spaces. Let $T : V \rightarrow W$ be a linear mapping from V to W.

(1) (5 points) What is a basis for a vector space?

(2) (5 points) What are the definitions of eigenvalues of eigenvectors of T?

(3) (10 points) Prove that the matrix equation $Tx = y$ has a solution if and only if $rank(T|y) = rank(T)$.

Problem 2. (Totally, 30 points)

Let V be a finite-dimensional inner product space over F, and let T be a linear operator on V. Denote the inner product of x and y by $< x, y >$, $\forall x, y \in V$. Denote the adjoint of T by T^*. Namely, $< T(x), y > = < x, T^*(y) >$, $\forall x, y \in V$. Suppose $TT^* = T^*T$. Denote the norm of x by $||x||$, $\forall x \in V$.

(1) (5 points) Prove that $||T(x)|| = ||T^*(x)||$, $\forall x \in V$.

(2) (5 points) Prove that if λ_1 and λ_2 are distinct eigenvalues of T with corresponding eigenvectors x_1 and x_2, then x_1 and x_2 are orthogonal.

(3) (10 points) Suppose $T = T^*$. Prove that every eigenvalue of T is real.

(4) (10 points) Let $g : V \rightarrow F$ be a linear transformation. Prove that there exists a unique vector $y \in V$ such that $g(x) = < x, y >$ for all $x \in V$.

Problem 3. (Totally, 20 points)

(1) (10 points) Let n be a natural number and $p \in (0, 1)$. Let X be a binomial random variable such that $P(X = i) = C_i^n p^i (1-p)^{n-i}$, $\forall i \in \{0, 1, 2, .., n\}$. Calculate the mean of X.

(2) (10 points) The moment generating function $\phi(t)$ of the random variable X is defined for all values t by $\phi(t) = E[e^{tX}]$. Calculate the moment generating function for an exponentially distributed random variable with mean λ.

Problem 4. (Totally 30 points)

Let $U_1, U_2, U_3, ...$ be a sequence of independent uniform $(0, 1)$ random variables. Namely, $\forall n \geq 1$, U_n is a continuous random variable and is uniformly distributed between zero and one. Define

$$N = \min\{n : n \geq 2, U_n > U_{n-1}\} \tag{1}$$

In other words, N is the index of the first uniform random variable that is larger than its immediate predecessor.

Define

$$M = \min\{n : n \geq 1, U_1 + U_2 + .. + U_n > 1\} \tag{2}$$

Namely, M is the number of uniform random variables whose sum we need to exceed one.

(1) (10 points) Derive the probability density function of N. (Hints: Consider the total number of ordering of $U_1, U_2, .., U_n$.)

(2) (5 points) Calculate the expected value of N.

(3) (15 points) Prove that the probability density function of M is identical to the probability density function of N. (Hints: Let $M(x) = \min\{n : n \geq 1, U_1 + U_2 + .. + U_n > x\}$. Show that $P\{M(x) > n\} = \frac{x^n}{n!}$ by conditioning on U_1.)

1. [20] Fourier Transform
   - A. [5] Find the Fourier transform of the single-sided exponential pulse $e^{-at}u(t)$ where $a>0$ and $u(t)$ is a unit step function.
   - B. [5] Find the Fourier transform of a two-sided exponential pulse defined by: $e^{-a|t|}$ where $a>0$.
   - C. [5] Find the Fourier transform of a time-shifted version of two-sided exponential pulse defined by: $e^{-a|t-t_0|}$ where $a>0$.
   - D. [5] Find the Fourier transform of a unit step function $u(t)$.
2. [20] Phase-Shift Keying
   - A. [10] In a coherent binary PSK system, the pair of signals $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by:

     $$s_1(t)=\sqrt{E_b}\phi_1 \text{ and } s_2(t)=-\sqrt{E_b}\phi_1$$

     where E_b is the transmitted signal energy per bit. If the signal is corrupted by an additive white Gaussian noise (AWGN) with zero mean and variance of $N_0/2$, find the corresponding bit error rate in terms of Q-function, which is defined by:

     $$Q(a)=\frac{1}{\sqrt{2\pi}}\int_a^{\infty}e^{-x^2/2}dx.$$
   - B. [5] Find the bit error rate of a QPSK system.
   - C. [5] Find the symbol error rate of a QPSK system.
3. [20] Matched Filter

   Prove that if a signal $s(t)$ is corrupted by AWGN, the filter with an impulse response matched to $s(t)$ maximizes the output signal-to-noise ratio. The maximum SNR obtained with the matched filter is:

   $$\text{SNR}_0=\frac{2}{N_0}\int_0^T s^2(t)dt=\frac{2\varepsilon}{N_0}$$
4. [20] Decision Rules
   - A. [10] Describe the "maximum a posteriori decision rule" and "maximum likelihood decision rule".
   - B. [10] If a binary information sequence $\mathbf{u}=(u_1, u_2, \ldots, u_n)$ is transmitted through an AWGN channel, prove that the maximum likelihood decision leads to finding the sequence with minimum Euclidean distance.
5. [20] Explanations:
   - A. [5] What is a wide sense stationary (WSS) process?
   - B. [5] What is an ergodic process?
   - C. [5] What is a cyclostationary process?
   - D. [5] Describe the OQPSK modulation scheme.

國立中山大學九十三學年度碩士班招生考試試題

科目：微分方程及向量分析（通訊所）（乙組）

Fill in the underlined blanks. Write you answers in the answer sheet. The detailed derivation is **NOT** required. (填空題，將答案寫在答案卷上，不須推導)

In the following $\vec{r} = \vec{a}_x x + \vec{a}_y y + \vec{a}_z z$ is the position vector, $r = |\vec{r}|$, and $\vec{a}_\alpha$ denotes the unit vector in the corresponding α direction.

1. (20pts) Consider a 2-D scalar field $V(x,y) = \dfrac{1}{(x-1)^2+1} + \dfrac{1}{(y-1)^2+1}$. The greatest directional derivative of V at the **origin** is __(1)__ and in the direction __(2)__ (specified using a unit vector). At what points does this greatest directional derivative reach maximum? Answer __(3)__. The positive maximum value is __(4)__.

2. (10pts) Let $\vec{A}(x,y,z) = \vec{a}_x xy - \vec{a}_y z^2 + \vec{a}_z x^2$, $V(x,y,z) = xyz$, and C is the curve $x = t^2$, $y = 2t$, $z = t^3$, evaluate the line integrals from $t=0$ to $t=1$ $\int_C \vec{A} \times d\vec{r} =$ __(5)__ and $\int_C V(x,y,z)\,d\vec{r} =$ __(6)__.

3. (20pts) Let field $\vec{A} = \vec{a}_r r^5$ exist in a ball of radius 1.

   (a) The divergence $\nabla \cdot \vec{A}$ in Cartesian coordinates is __(7)__.

   (b) The volume integral $\int_V \nabla \cdot \vec{A}\,dv$ over the unit-radius ball is __(8)__.

   (c) The surface integral $\int \vec{A} \cdot d\vec{s}$ over the surface of the top-half hemisphere of the unit-radius ball is __(9)__.

   (d) The surface integral $\int \vec{A} \cdot d\vec{s}$ over the xy-plane, with $d\vec{s}$ in the positive z direction, is __(10)__.

4. (20pts) We wish to solve the differential equation $y'' - 2y' + y = e^x / x^3$, $x > 0$.

   (a) Find the homogeneous solution $y_h(x) =$ __(11)__.

   (b) Find a particular solution $y_p(x) =$ __(12)__.

   (c) Given the initial conditions $y(1) = 0$, $y'(1) = 0$, we can find the complete solution $y(x)$. Find $y(\ln 2) =$ __(13)__ and $y(2) =$ __(14)__.

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科目：微分方程及向量分析（通訊所）（乙組）

5. (15pts) The differential equation and initial conditions are given by $y'' - 3y' + 4y = 0$, and $y(0) = 1$, $y'(0) = 5$

   (a) Find the Laplace transform $Y(s)$ of $y(x)$. $Y(s)$ =___(15)___.

   (b) Use the inverse Laplace transform to solve for $y(x)$. Write the solution as $y(x) = e^{ax} A\cos(\omega x - \phi)$, where $A > 0$. Then a =___(16)___ and A =___(17)___.

(15pts)

6. Consider the Sturm-Liouville problem $y'' + \lambda y = 0$; $y(0) = 0$, $y'(\pi) = 0$. Suppose the eigenvalues and the corresponding eigenfunctions are respectively λ_n and y_n, $n = 1, 2, \cdots$, where $0 < \lambda_1 < \lambda_2 < \cdots$. Find

   (a) λ_1 =___(18)___, (b) λ_2 =___(19)___, (c) $y_4(\pi) / y_4(\pi/2)$ =___(20)___.

Electromagnetics

May 2, 2004

1. Find the input impedance of a low-loss quarter-wavelength line (20%)
   a. Terminated in a short circuit.
   b. Terminated in an open circuit.

2. Consider two point charges of Q1 = +1μC and Q2 = +2μC located respectively at (1, 0)m and (-1, 0)m. (20%)
   a. What is the magnitude and direction of the electrical force felt by a third charge Q3 = +1nC when placed at (0, 1)m?
   b. At what point(s) must the third charge Q3 = +1nC be placed in order to experience no net force?

3. Two circular coils with centers on a common axis have N1 and N2 turns, each of which is closely wounded, and radii a and b, respectively. The two coils are separated by a distance d, which is assumed to be much larger than both radii (i.e., $d \gg a, b$). Please find the mutual inductance between the coils. (20%)

4. Consider a certain type of humid soil with the following properties: $\sigma \approx 10^{-2}\,\text{S-m}^{-1}$, $\varepsilon_r = 30$, $\mu_r = 1$. Find the ratio of the amplitudes of the conduction and displacement currents at 1 kHz, 1 MHz, and 1GHz. (20%)

5. An air-filled parallel-plate waveguide has a plate separation of 1.25cm. Find (a) the cutoff frequencies of the TM_0, TE_1, TM_1, TM_2. (b) The phase velocities of those modes at 15GHz. (20%)