1. (15%) For the following matrices

$$A=\begin{bmatrix}4 & -1 & 0 & -1\\ 1 & 4 & 0 & 0\\ 0 & 0 & 4 & 1\\ 2 & 0 & -1 & 4\end{bmatrix}, B=\begin{bmatrix}4 & -1 & 0 & 0\\ -1 & 4 & 0 & 0\\ 0 & 0 & 4 & -1\\ 0 & 0 & -1 & 4\end{bmatrix}, C=\begin{bmatrix}4 & -1 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 4 & -1\\ 0 & 0 & 0 & 4\end{bmatrix}$$

(a) (5%) Determine whether they are orthogonally diagonalizable.

(b) (10%) Find the orthogonal matrices that diagonalize them if they are orthogonally diagonalizable.

2. (5%) Find a matrix A for the linear operator $T:\Re^3 \to \Re^3$ that first rotates a vector counterclockwise about the z-axis though an angle $60°$, then reflects the resulting vector about the yz-plane, and then projects that vector orthogonally onto the xy-plane.

3. (10%) Find QR-decomposition of

$$A=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 3 & 6\\ 0 & 0 & 2 & 4\end{bmatrix}.$$

4. (10%) Find the orthogonal projection of the vector $u=(-1,0,1,3)$ on the subspace of $\Re^4$ spanned by the vectors
$u_1=(3,1,0,2)$, $u_2=(3,6,3,3)$, $u_3=(-2,0,4,-2)$.

5. (10%) Given vectors $u=(1,0,1)$, $v=(1,3,2)$, $w=(0,5,3)$, solve each of the following

(a) (5%) $u\times(v\times w)$.

(b) (5%) $\|w\|^2 u+\|u\|^2 v$.

6. (20%) Consider the following matrix

$$A=\begin{bmatrix}1 & -1 & 1\\ -1 & 2 & 1\\ 3 & -1 & 2\end{bmatrix}.$$

(a)(10%) Find an LU decomposition of the matrix.

(b)(10%) Use LU decomposition to solve the system

$$\begin{aligned} x_1 - x_2 + x_3 &= 4 \\ -x_1 + 2x_2 + x_3 &= -1. \\ 3x_1 - x_2 + 2x_3 &= 8 \end{aligned}$$

7. (20%) Suppose that $T:\Re^2 \to \Re^3$ is defined by

$$T\left(\begin{bmatrix}x_1\\ x_2\end{bmatrix}\right)=\begin{bmatrix}3x_1-4x_2\\ -x_1+2x_2\\ 5x_1\end{bmatrix}.$$

(a) (7%) Determine a spanning set for the range of T.

(b) (7%) Determine a spanning set for the null space of T.

(c) (2%) Is T onto?

(d) (2%) Is T one-to-one?

(e) (2%) Is T invertible?

8. (10%) Let T be a linear operator on $\Re^3$ such that

$$T\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\2\\-1\end{bmatrix},\quad T\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right)=\begin{bmatrix}3\\-1\\1\end{bmatrix},\quad T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}2\\0\\3\end{bmatrix}.$$

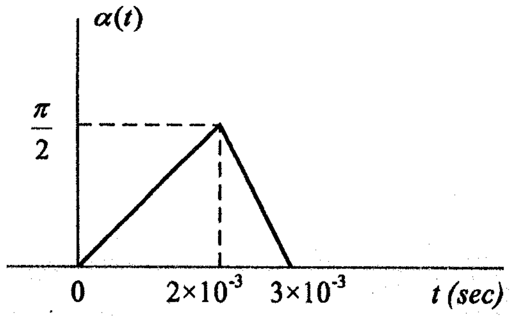
Find the standard matrix of T.

通訊理論 (Communications Theory)

1. (15%) Let $x(t) = t^{-0.25}$, $t \geq t_0 > 0$, and zero otherwise. Compute the energy and power in $x(t)$, and determine whether $x(t)$ is an energy-type signal or a power-type signal.

2. (15%) Find the Hilbert Transform $\widehat{x}(t)$ of a signal $x(t) = \cos(\omega_0 t) + \sin(\omega_0 t)$. Based on your result, determine whether $\widehat{x}(t)$ and $x(t)$ are orthogonal.

3. (15%) Consider the angle modulated wave $cos(\omega_c t + \alpha(t))$, where $\alpha(t)$ is shown below.



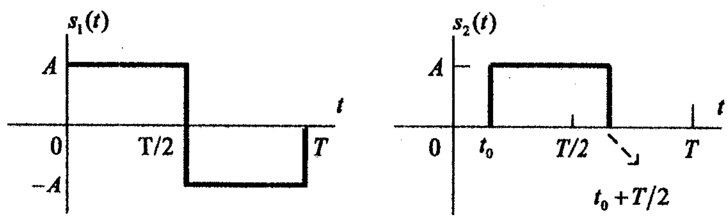
   (a) (5%) What is the maximum frequency deviation?

   (b) (10%) Now consider the composite wave

$$e(t) = cos\omega_c t + cos(\omega_c t + \alpha(t)).$$

   Let $e(t) = \text{Re}\{a(t)e^{j\phi(t)}e^{j\omega_c t}\}$. Draw the locus of $a(t)e^{j\phi(t)}$ for the time interval $0 \leq t \leq 3 \times 10^{-3}$ seconds. What is the maximum of the phase deviation $\phi(t)$?

4. (20%) A pair of pulses are shown below.



   (a) (6%) Find the optimum (matched) filter impulse response $h_0(t)$ for $s_1(t)$ and $s_2(t)$.

(b) (8%) What is the best choice for t_0 such that the error probability at the receiver is minimized? Why?

(c) (6%) Sketch a correlator receiver structure for these signals.

5. (20%)

(a) (6%) What are the differences between source coding and channel coding?

(b) (6%) What is Sampling Theorem? Describe the theorem, applications and drawbacks.

(c) (8%) List two types of degradation from which the error performance of digital signaling suffers. What are the typical solutions to dealing with the error sources?

6. (15%) Twenty-five audio input signals, each bandlimited to $3.5kHz$ and sampled at a $10kHz$ rate, are time-multiplexed in a PAM system.

(a) (7%) Determine the minimum clock frequency of the system.

(b) (8%) Find the maximum pulse width for each channel.

~End~

1. Let X_1 and X_2 be two continuous random variables with joint probability density function

$$f_{X_1X_2}(x_1,x_2)=\begin{cases}4x_1x_2, & if \quad 0<x_1<1, \quad 0<x_2<1\\ 0 \;, & otherwise\end{cases}$$

(a) Find $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$. (10%)

(b) Find the joint probability density function of Y_1 and Y_2 , $f_{Y_1Y_2}(y_1, y_2)$, where $Y_1 = X_1^2$ and $Y_2 = X_1X_2$, (8%)

(b) Find $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$ (7%)

2. Consider a communication channel corrupted by noise. Let random variable X be the value of the transmitted signal and Y be the value of the received signal. Assume that the conditional density of Y is given $\{X = x\}$ is Gaussian, i.e.

$$f_{Y|X}(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(y-x)^2}{2\sigma^2}\right]$$

and X is uniformly distributed on [-1, 1].

(a) What is the probability density function of Y, $f_Y(y)$ (7%)

(b) What is the conditional density of X is given Y (*i.e.* $f_{X|Y}(x|y)$)? (8%)

3. A zero-mean normal (*Gaussian*) random vector $\mathbf{X} = (X_1, X_2)^T$ has *covariance matrix* $\mathbf{K} = E[\mathbf{X}\mathbf{X}^T]$, which is given by

$$\mathbf{K} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \qquad (10\%).$$

Find a transformation $\mathbf{Y}=\mathbf{D}\mathbf{X}$ such that $\mathbf{Y}= (Y_1, Y_2)^T$ is a normal (or Gaussian) random vector with uncorrelated (and therefore *independent*) components of unity variance

4. Consider two *independent identical distribution* (*i.i.d*) *random variables*, X and Y.

(a) Find the *probability density function* of *random variable*, Z=X+Y. (7%)

Now, if the *probability density functions* of X and Y are with

$$f_X(x) = f_Y(x) = \frac{1}{a}rect\left(\frac{x}{a}\right) = \begin{cases}\frac{1}{a} & ,-\frac{a}{2} \le x \le \frac{a}{2}\\ 0 & , \quad otherwise\end{cases}$$

(b) Find the *Characteristic function* of X and Z, where $\Phi_X(\omega) = E[e^{j\omega X}]$. (8%)

(c) Compute the *probability density function* of Z. (5%)

5. Let X_1 and X_2 be two independent *Poisson* random variables with identical distribution.

(a) Find P[$X_1 = x_1 \mid X_1 + X_2 = y$] (8%)

(b) Find E[$X_1 \mid X_1 + X_2 = y$] (7%)

6. Assume that *random variable* X has a *gamma distribution* with *probability density function*, which is defined by

$$f_X(x) = \begin{cases} \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$. The *gamma function* $\Gamma(\alpha)$ is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$

for $\alpha > 0$ and has the following properties, e.g., $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$, $\Gamma(n+1) = n!$, $\Gamma(1/2) = (\pi)^{1/2}$ and $\Gamma(1) = 1$.

(a) For $\alpha = 1/2$ and $\beta = 2$, please evaluate the *mean* $\mu = E[X]$, and *variance* $\sigma_X^2 = E[(X - E[X])^2]$ (8%).

(b) For $\alpha = \upsilon/2$ and $\beta = 2$, we have the so-called *chi-square distribution*, again, find the *mean* μ and σ_X^2 for *random variable* X (7%).

(*Note:* υ is the degree of freedom and $\upsilon > 0$)

1. A spherical dielectric shell of an inner radius r_i and an outer radius r_o is centered at the origin and has a dielectric constant of ε_r. Given a charge distribution

$$\rho_v \text{ (C/m}^3) = \begin{cases} \rho_0(1 - r^2/r_i^2) & r < r_i \\ 0 & \text{else} \end{cases}, \text{ where } r = \sqrt{x^2+y^2+z^2}, \text{ determine} \qquad (20\%)$$

(a) $\vec{E}$ in $0 \le r < r_o$, (10%)

(b) V and $\vec{P}$ inside the dielectric shell. (10%)

2. (20pts) An air coaxial line with the z-axis as its axis has a hollow inner conductor of radius a and a very thin outer conductor of radius b. Assume a current I flows in the inner conductor and returns in the outer conductor. Denote $\rho = \sqrt{x^2+y^2}$. Calculate (20%)

(a) the magnetic flux density B in $\rho < a$ and $a < \rho < b$, respectively, (10%)

(c) the magnetic energy per unit length stored in the line, (5%)

(d) the inductance per unit length. (5%)

3. Determine the polarization of the following electric fields: (4% each)

(a) $\mathbf{E} = \mathbf{a}_z E_0 \cos(\omega t - \beta y) + \mathbf{a}_x E_0 \sin(\omega t - \beta y)$

(b) $\mathbf{E} = \mathbf{a}_y E_0 \cos(\omega t + \beta x) + \mathbf{a}_z E_0 \sin(\omega t + \beta x)$

(c) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) - \mathbf{a}_z E_0 \sin(\omega t + \beta y)$

(d) $\mathbf{E} = \mathbf{a}_z E_0 \cos(\omega t - \beta x) - \mathbf{a}_y E_0 \sin(\omega t - \beta x + \pi/4)$

(e) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) + \mathbf{a}_z E_0 \cos(\omega t - \beta y)$

4. Consider the partially-filled parallel plate waveguide shown in Fig. P.4. Derive the expressions of electric and magnetic fields inside the waveguide and the cutoff frequency for the TM modes. Can a TEM wave exist in the structure? Ignore fringing fields at the sides with $w >> d$. (20%)

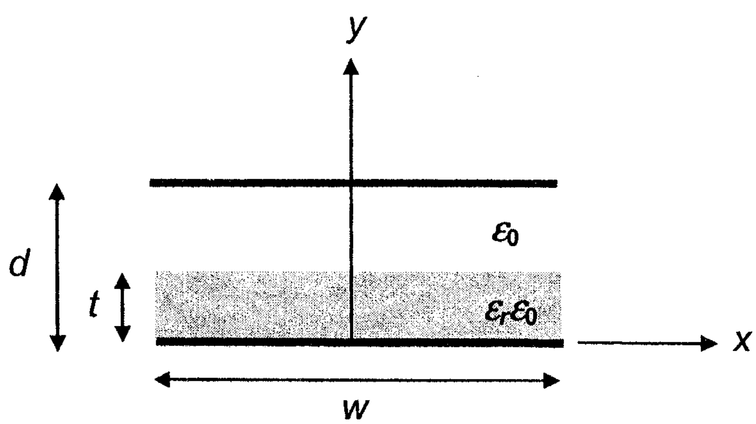


Fig. P4.

5. Consider the quarter-wave impedance matching circuit shown in Fig. P5. Derive the expressions for the amplitude of forward and reverse traveling waves on the quarter-wave line section, V^+ and V^-, in terms of the amplitude of the incident voltage, V_i. (20%)

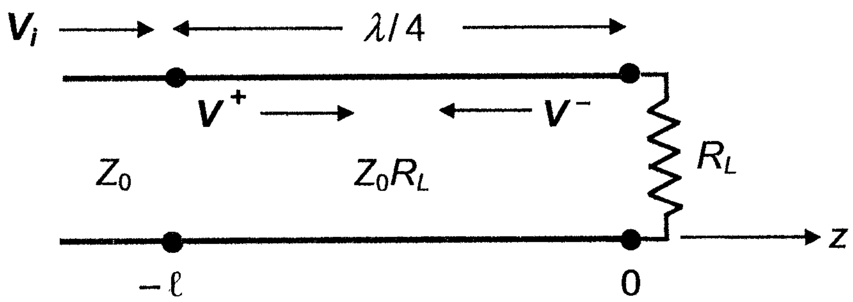


Fig. P5.

1. (10%) Solve the initial value problem (IVP) for the following ODE

$$y'' + y = 5x + 8\sin x, \quad y(\pi) = 0,\ y'(\pi) = 2.$$

2. (15%) Find the solution of the initial value problem

$$y'' + 2y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

3. (5%) Find the Laplace transform of (a) $\mathcal{L}\{e^{-5t}\}$ (b) $\mathcal{L}\{\sin 3t\}$

4. (15%) Solve the initial value problem by the Laplace transform

$$\begin{cases} y_1' + 2y_2' = 1 \\ 3y_1' + y_2' + y_2 = t \end{cases} \quad y_1(0) = 0,\ y_2(0) = 0$$

5. (15%) Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 2, & 0 \le x < \pi \end{cases}$ in a Fourier series.

6. (13%) Evaluate the following integral

$$\oint_C \frac{dz}{\sinh\ (2z)},$$

where z is a complex variable and C denotes the circle $|z| = 2$ described in the positive sense.

7. (15%) The set

$$S = \left\{ \frac{1}{\sqrt{2}},\ \cos\ x,\ \cos\ 2x,\ \cos\ 3x,\ \cos\ 4x \right\}$$

is an orthonormal set of vectors in $C[-\pi, \pi]$ with inner product defined as

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) g(x)\ dx,$$

where $C[-\pi, \pi]$ is the set of all functions f that are continuous on $[-\pi, \pi]$. Suppose that the function $\sin^4 x$ can be written in a linear combination of elements of S as

$$\sin^4 x = \frac{3\sqrt{2}}{8}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}(\cos\ 2x) + \frac{1}{8}(\cos\ 4x).$$

Use the above equation and orthogonal basis property (but do not compute antiderivatives, otherwise you will get zero credit), find the values of the following integrals:

(i) $\int_{-\pi}^{\pi} \sin^4 x\ dx$ (ii) $\int_{-\pi}^{\pi} \sin^4 x \cos(3x)\ dx$ (iii) $\int_{-\pi}^{\pi} \sin^4 x \cos(4x)\ dx$

8. (12%) Let P_4 be the set of all polynomials of degree less than 4. In P_4 the inner product is defined by

$$\langle p, q \rangle = \sum_{i=1}^{4} p(x_i) q(x_i),$$

where $x_i = (i-2)/2$ for $i = 1, \cdots, 4$. Its norm is defined by

$$\| p \| = \sqrt{\langle p, q \rangle} = \left\{ \sum_{i=1}^{4} [p(x_i)]^2 \right\}^{1/2}.$$

Compute (a) $\| x^2 \|$, (b) the distance between x and x^2.

1. (10%) Design a non-inverting amplifier with a gain of 1.5 V/V using three 100 kΩ resistors.

2. (10%) A 1-mA diode having a 0.1 V/decade characteristic operates from a constant-current supply with $V_D = 0.8$ V. If it is shunted by two more identical diodes, what does the voltage drop become?

3. (10%) For the circuit shown in Fig. 1, find I_C and V_{CE} for $V_{BE} = 0.7$ V and $\beta = 50$.

4. (10%) For the FET circuit shown in Fig. 2, I_{DSS}=4 mA and $V_P = -2$ V. Find I_D and V_o.

5. (10%) For the circuit shown in Fig. 3, $R_1 = R_2 = 10$ kΩ and $C_1 = C_2 = 100$pF. Find the upper 3-dB frequency exactly.

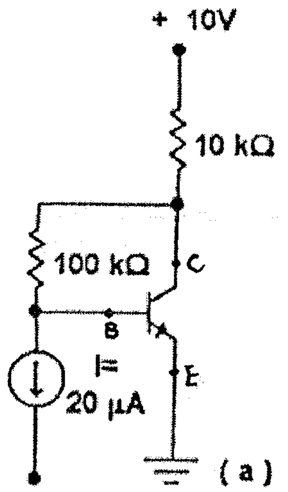


Figure 1

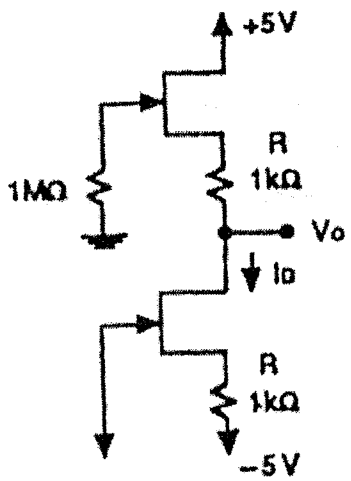


Figure 2

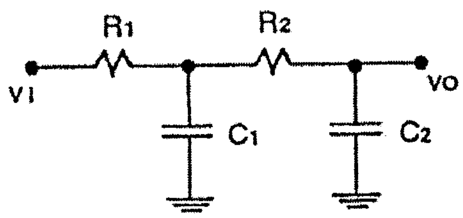


Figure 3

6. (20%) The feedback amplifier in Figure 4 has I = 1mA and V_{GS} = 0.8 V. The MOSFET has V_t= 0.6V and V_A = 30 V. For R_s = 10 kΩ, and R_1 = 1 MΩ, and R_2 = 4.7 MΩ, find (a) the feed-back configuration, (b) the voltage gain v_o / v_s, (c) the input resistance R_{in}, and (d) the output resistance R_{out}.

7. (10%) Using a simple (r_π, g_m) model for each of the two transistors Q_{18} and Q_{19} in Figure 5, find the small-signal resistance between A and A' assuming I_{C18} = 165 μA and I_{C19} = 16μA.

8. (20%) Figure 6 shows the circuit for determining the op-amp output resistance when v_o is positive and Q_{14} is conducting most of the current. Using the resistance of the Q_{18}-Q_{19} network calculated in Figure 5 and neglecting the large output resistance of Q_{13A}, find R_{out} when Q_{14} is sourcing an output current of 5 mA.

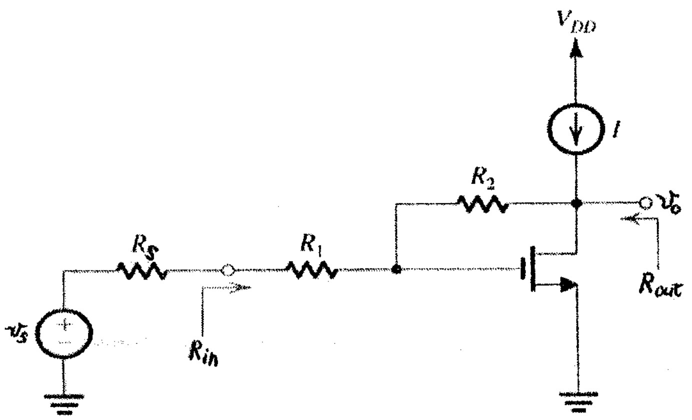


Figure 4

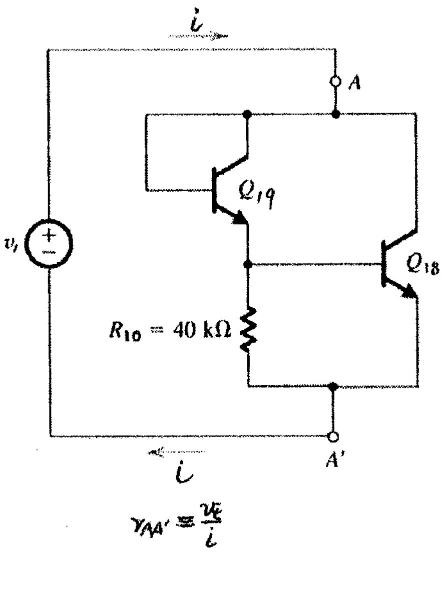


Figure 5

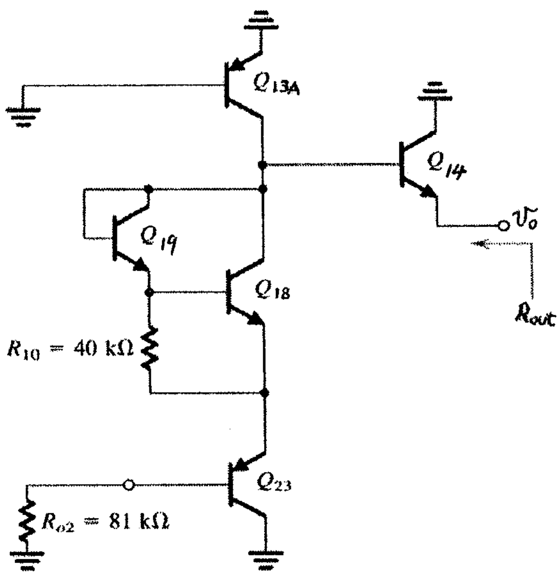


Figure 6