國立中山大學九十一學年度碩士班招生考試試題

科目：工程數學（甲）[電機系碩士班]（甲、丙選、丁、戊組） 共 1 頁 第 1 頁

1. (10%) We would like to evaluate an integral involving the derivative of the Dirac δ-function.

   (a) Consider $I=\int_{-\infty}^{\infty}\cos t\,\delta'(t-\frac{\pi}{2})\,dt$. Let $u(t)=\delta(t-\frac{\pi}{2})$. Then $I=\int_{t=-\infty}^{\infty}\cos t\,du$.

   Make use of integration by part to obtain the answer.

   (b) Find the a general formula for $\int_{-\infty}^{\infty}x(t)\delta'(t-t_0)\,dt$.

2. (15%) Let $x(t)$ be a rectangular pulse defined by $x(t)=1$, $|t|<1/2$ and $x(t)=0$, otherwise. The corresponding Fourier transform is denoted as $X(j\omega)$, i.e.,

   $X(j\omega)=\int_{-\infty}^{\infty}x(t)\exp(-j\omega t)\,dt$. Calculate

   (a) $X(j\omega)$, (b) $\int_{-\infty}^{\infty}\frac{\sin\omega}{\omega}d\omega$, (c) $\int_{-\infty}^{\infty}\frac{\sin\omega\cos(2\omega)}{\omega}d\omega$

3. (10%) Let $\vec{F}=\vec{a}_x 2xy+\vec{a}_y x^2+\vec{a}_z(z-1)$. Evaluate the line integral $\int_{(0,0,0)}^{(1,1,0)}\vec{F}\cdot d\vec{\ell}$ along a parabola $y=x^2$ on the xy plane.

4. (14%)

   (a) (6%) Let $A\in\mathbb{R}^{m\times n}$. Show that: $N(A^TA)=N(A)$.

   (b) (8%) Let U and V be subspaces of a vector space W and suppose $W=U+V$. Show that $U\bigcap V=\{0\}$ if and only if for any $w\in W$, there exist a unique $u\in U$ and a unique $v\in V$ such that $w=u+v$.

5. (16%) Let $A\in\mathbb{R}^{n\times n}$ have no eigenvalues being 1 and -1.

   (a) (8%) Show that $A^T=-A$ if and only if e^{At} is an orthogonal matrix for all t.

   (b) (8%) Show that $A^T=-A$ if and only if $(I-A)(I+A)^{-1}$ is an orthogonal matrix.

6. (15%) Find the general solution of the following differential equation:

   $6xy\,dx+(4y+9x^2)\,dy=0$

7. (20%) Use the Laplace transform to solve the following initial value problem:

   $y''+2y'+2y=\sin t$, $y(0)=1$, $y'(0)=1$.

(1). Design a Wien-bridge circuit to oscillate at a specified frequency f_o = 20 kHz. You need draw the Wien-brige Circuit (5%) and analyze it (7%), then decide the values of capacitors and resistors (8%).

(2). In Figure 2 the small signal equivalent of a bipolar junction transistor with simplified hybrid – π model and including the equivalent Miller capacitance are shown respectively, where the circuit parameters are $R_C = R_L = 4\,\mathrm{k}\Omega$, $r_\pi = 2.6\,\mathrm{k}\Omega$, $R_B = 200\,\mathrm{k}\Omega$, $C_\pi = 4$ pF, $C_\mu = 0.2$ pF, and $g_m = 38.5$ mA/V. Please determine the 3 dB frequency of the current gain for the circuit both with and without the effect of C_M. (20%)

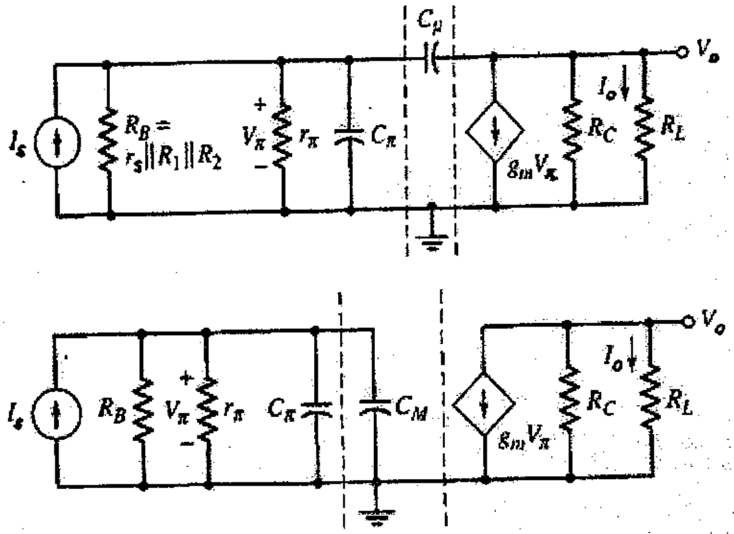


Figure 2

(3).

(a) For the circuit in Figure 3, find I_{O1} and I_{O2} in terms of I_{REF}. Assume all transistors to be matched with current gain β. (10%)

(b) Use this idea to design a circuit that generates currents of 1, 2, and 4 mA using a reference current source of 7mA. What are the actual values of the currents generated for β = 50? (10%)

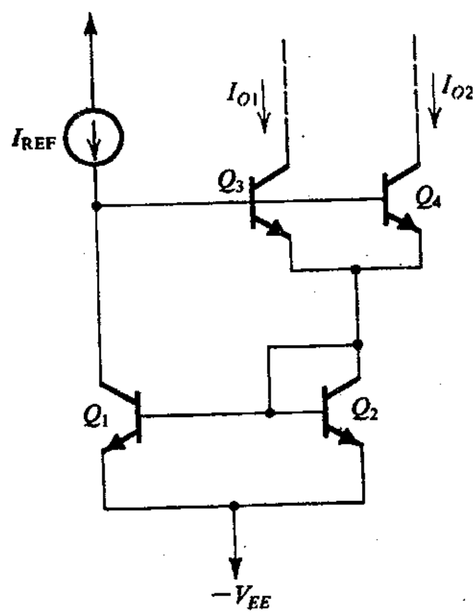


Figure 3

(4).

(a) What is the circuit shown in Figure 4? (4%)

(b) What is the maximum positive output voltage v_{omax} of the circuit and what situation will limit this value? (8%)

(c) What is the minimum output voltage (i.e. maximum negative amplitude) v_{omin} of the circuit and what situation will limit this value? (8%)

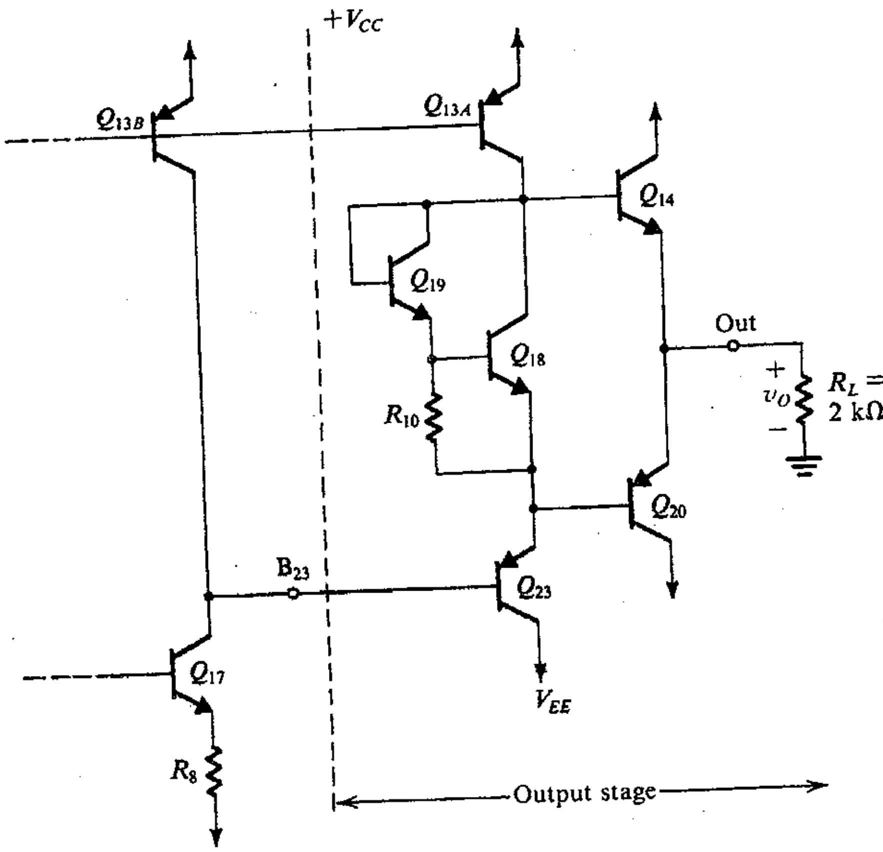


Figure 4

(5).

(a) Draw the current flow in a *pnp* BJT transistor biased to operate in the active mode and explain the Early effect with respect to α and β. (10%)

(b) Draw the current flow in a PMOS transistor biased to operated in saturation region and explain the channel length modulation effect. (10%)

科目：半導体概論（電機系）（甲組）

1. A direct semiconductor with $n_i = 1.5 \times 10^{10} cm^{-3}$ is doped with $N_d = 10^{15} cm^{-3}$. Its low level carrier lifetime is $\tau_n = \tau_p = 10^{-7}$s. The sample is uniformly exposed to a steady optical generation rate of $g_{op} = 2 \times 10^{22}$ EHP/cm^3-s. For this excitation, calculate the excess carrier concentration Δn.(hint: this is not a low level injection) (17%)

2. Assume the critical field for breakdown is 5×10^5 V/cm, calculate the breakdown voltage in the following abrupt Si junction: $N_a = N_d = 1 \times 10^{18} cm^{-3}$. (note: $\epsilon_{Si} = 11.8\ \epsilon_o, \epsilon_o = 8.85 \times 10^{-14}\ F/cm^2$, $n_i = 1.5 \times 10^{10} cm^{-3}$) (17%)

3. An Au-n-GaAs Schottky Contact is at T=300K^o with $N_d = 5 \times 10^{15} cm^{-3}$ (ϕ_m=5.1 Volt, χ=4.07 Volt, $N_c = 4.7 \times 10^{17} cm^{-3}$, $\epsilon = 13.1\epsilon_o$). Calculate (17%)

   (a) the build in voltage V_{bi}.

   (b) the barrier lowering $\Delta\phi$ at zero bias voltage.

4. Describe and explain the I-V characteristics of a thyristor (semiconductor contorlled rectifier, SCR). (16%)

5. Consider the npn BJT that is switched from saturation to cut-off. The base current is switched from I_B to zero. Derive the expression of the turn-off storage delay time t_{sd}. (16%)

6. Consider a silicon solar cell of area 2 cm^2. If the dopings of the solar cell are $N_a = 1.7 \times 10^{16} cm^{-3}$ and $N_d = 5 \times 10^{19} cm^{-3}$, and given $\tau_n = 10\mu s$, $\tau_p = 0.5\mu s$, $D_n = 9.3\ cm^2/s$, $D_p = 2.5\ cm^2/s$. The short circuit current is $I_{sc} = 100$ mA under the sun light. Determine the open circuit voltage V_{oc}. (note: $\epsilon_{Si} = 11.8\ \epsilon_o, \epsilon_o = 8.85 \times 10^{-14}\ F/cm^2$, $n_i = 1.5 \times 10^{10} cm^{-3}$)(17%)

1. (15%) Let $x(t)$ be a rectangular pulse defined by $x(t)=1$, $|t|<1/2$ and $x(t)=0$, otherwise. The corresponding Fourier transform is denoted as $X(j\omega)$, i.e.,

$X(j\omega)=\int_{-\infty}^{\infty} x(t)\exp(-j\omega t)\,dt$. Calculate

(a) $X(j\omega)$, (b) $\int_{-\infty}^{\infty}\frac{\sin\omega}{\omega}d\omega$, (c) $\int_{-\infty}^{\infty}\frac{\sin\omega\cos(2\omega)}{\omega}d\omega$

2. (10%) Let $\vec{F}=\vec{a}_x 2xy+\vec{a}_y x^2+\vec{a}_z(z-1)$. Evaluate the line integral $\int_{(0,0,0)}^{(1,1,0)}\vec{F}\cdot d\vec{\ell}$ along a parabola $y=x^2$ on the xy plane.

3. (16%)

(a) (8%) Let $A\in\mathbb{R}^{m\times n}$. Show that: $N(A^TA)=N(A)$.

(b) (8%) Let U and V be subspaces of a vector space W and suppose $W=U+V$. Show that $U\cap V=\{0\}$ if and only if for any $w\in W$, there exist a unique $u\in U$ and a unique $v\in V$ such that $w=u+v$.

4. (24%)

(a) (7%) Let $A\in\mathbb{R}^{n\times n}$. Show that A is skew symmetric if and only if e^{At} is an orthogonal matrix for all t.

(b) (6%) Consider the dynamic system

$$\dot{x}(t)=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}x(t) \quad \text{with} \quad x(0)=\begin{pmatrix}\alpha\\ \beta\end{pmatrix}\neq\begin{pmatrix}0\\ 0\end{pmatrix}.$$

Use *similar transformation technique* to *diagonalize* the system matrix, then solve $x(t)$.

(c) (6%) (i) Find the size of $x(t)$ in the normed space $(\mathbb{R}^2,\|\cdot\|_2)$ for all $t\geq 0$.

(ii) Find all $t\geq 0$ such that $x(t)$ is orthogonal to $x(0)$.

(d) (5%) Explain the results of (c) from exploiting the property proved in (a).

5. (15%) Find the general solution of the following differential equation:

$$6xy\,dx + (4y + 9x^2)\,dy = 0$$

6. (20%) Use the Laplace transform to solve the following initial value problem:

$$y'' + 2y' + 2y = \sin t \quad , \; y(0) = 1 \quad , \; y'(0) = 1.$$

1.(24%) Explain or answer the following terminologies:

(a) asymptotic stability (b) state transition matrix and transfer functions (c) What is feedback control and what are its effects? (d) Nyquist plot and Nyquist stability (e) phase lead controller and effects of phase lead compensation (f) control system "type" and steady-state error.

2.(13%) A controlled process is modeled by the following state equations:

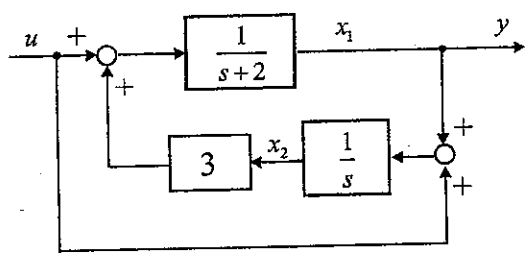
$$\dot{x}_1(t) = x_1(t) - 2x_2(t), \qquad \dot{x}_2(t) = 10x_1(t) + u(t)$$

The control $u(t)$ is obtained from state feedback, such that

$$u(t) = -k_1 x_1(t) - k_2 x_2(t),$$

where k_1 and k_2 are real constants. Determine the region in the k_1 versus k_2 parameter plane in which the closed-loop system is asymptotically stable.

3.(13%) Determine the controllability and observability of the system shown below by the following methods:



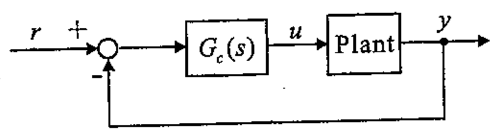
(a) Conditions on the **A**, **B**, **C**, and **D** matrices.

(b) Conditions on the pole-zero cancellation of the transfer functions.

4.(a)(6%) Explain why the root locus (for $0 \le k < \infty$) are found in the section of the real axis only if the total number of poles and zeros of $G(s)H(s)$ to the right of that section is odd.

(b)(6%) Suppose the relative degree of a minimum phase system is r. What is the slope of the asymptote of the magnitude curve of Bode plot as $\omega \to \infty$? What is the phase angle of the asymptote of the phase curve of Bode plot as $\omega \to \infty$?

5. Consider a system with an unstable plant shown as follows:



where $G_c(s)$ is the controller to be designed, and the state-space representation of the plant is

$$\begin{aligned} \dot{x} &= x + 3u, \\ y &= x + u. \end{aligned}$$

(a)(10%) Design a controller with the transfer function $G_c(s) = \dfrac{k}{s+\alpha}$ so that the damping ratio of the closed-loop system is 0.5, and the undamped natural frequency ω_n is 1 rad/sec.

(b)(6%) If the answer of α you get in part (a) is 4, find the range of k so that the system is stable.

(c)(6%) Find the phase margin when $k = \sqrt{2},\ \alpha = \sqrt{2}$.

6.(16%) Assume that $\mathbf{A} \in R^{n\times n}$, $\mathbf{b} \in R^{n\times 1}$, and $\mathbf{k} \in R^{1\times n}$. Prove that the system is controllable iff $\{\mathbf{A} - \mathbf{bk},\ \mathbf{b}\}$ is a controllable pair.

國立中山大學九十一學年度碩士班招生考試試題

科目：離散數學（電機系）（丙組選考） 共 1 頁 第 1 頁

（回答證明題與計算題需嚴謹，每個推導步驟均需寫出明確的原由）

1. (10%) Prove by induction that $11^{n+2} + 12^{2n+1}$ is dividable by 133 for all $n \geq 0$
2. (10%) Given 3 sets A, B, and C, prove that $(A - B) - C = (A - C) - (B - C)$
3. (10%) 學校辦園遊會，電機系設有 l 個攤位，每個攤位以小時爲單位需要 k 位同學照顧，園遊會共長 n 個小時，電機系派出 k*n 位同學協辦照顧攤位，每位同學負責 1 小時，請問共有多少種可能的排班方式?
4. (10%) Given a relation matrix R of 5 objects in a set {1, 2, 3, 4, 5},

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

- A value 1 of R matrix element (m,n) (i.e. row m and column n) means that object m relates to object n. A value 0 means that it does not relates to object n.
- For its transitive matrix R^1 $(=R)$, R^2, R^3, … , we can represent them as functions $f^1(m,n)$, $f^2(m,n)$, $f^3(m,n)$, … , where m and $n \in \{1,2,3,4,5\}$ and $f^i(m,n) \in \{0,1\}$.

Derive the values of $f^1(4,3)$, $f^2(5,1)$, $f^3(2,1)$, and $f^4(5,1)$.

5. (10%) Given boolean variables p, q, and r, (p', q', and r' represent the complement of p, q, and r, respectively), prove the formula is true:

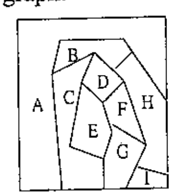
$$p \wedge q \rightarrow (r \wedge (p' \vee q)) \vee (p \wedge r')$$

6. (10%) Prove the following equation with generating function :

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

(Note: Prove in ways other than using generating function will not get any score)

7. (15%) For a complete graph K_n,
   (a) (10%) What kind of n value would the complete graph contains a Eularian circuit (歐拉迴路)? Prove your answer.
   (b) (5%) What kind of n value would the complete graph does not contain a Eularian circuit? Prove it.
8. (10%) Given a map (地圖) in the following,
   (a) (5%) Construct the adjacency (相鄰) graph of the map where each vertex represents a region in the map and each edge represents a border between two adjacent regions.
   (b) (5%) Partition the graph vertex set into a minimum number of disjoint vertex subsets such that between any two vertices in the same partition, there does not exist an edge in the adjacency graph.



9. (15%) A binary search tree is a rooted tree where each node is labeled a distinct value. It maintains a property that:
   - the values of all nodes in its left subtree is smaller than that of the root node
   - the values of all nodes in its right subtree is larger than that of the root node

   This property also holds for any subtree of the tree, recursively.
   Write a recursive algorithm to delete the node with its label matching a given value K.
   (i.e. After the node is deleted, the tree must still be a binary search tree.)
   (Hint: 先根據 node 在 tree 上所在位置關係做分類,再依各分類作 deletion 方法的敘述)

1. [6 分] 請將下列 prefix 和 postfix 的表示法分別轉換成 postfix 和 prefix 的表示法，並求其值，且用 stack 把每一計算步驟都表示出來。
   (a) /-2-*$238/+35*226
   (b) 25325-+$*53*-

2. [4 分] 下圖為一環狀 queue：

			front			rear	
0	1	2	3	4	5	6	7
				E	F	G	

   依序執行下列指令： 加入 A => 加入 D => 刪除 => 加入 B => 刪除 => 加入 C => 加入 H => 刪除 => 刪除，請畫出每一指令執行後，queue 之狀態及 front、rear 的位置。

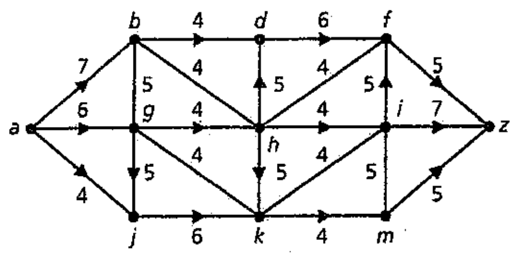
3. [32 分] 設有一數列為 395、701、572、402、636、518、590、613、655、347、111、100。請根據下列(a)-(g)之方法分別做由小到大之排序，並將每一步驟詳細畫出來。最後，請回答問題(h)：
   (a) Bubble sort。
   (b) Selection sort。
   (c) Insertion sort。
   (d) Shell sort。
   (e) Quicksort。
   (f) Merge sort。
   (g) Heapsort。
   (h) 上述各種方法之時間複雜度各為何？

4. [12 分] 設有一字母集合為 $\{a, z, b, y, c, x, d, w, e, v, f\}$，字母大小順序為 $a<b<\ldots<y<z$。請
   (a) 依其由左至右的順序建立一 AVL tree，並將每一步驟詳細畫出來。
   (b) 依 LIFO（Last key inserted is first removed）的順序原則將每個字母移除掉，並將每一步驟詳細畫出來。
   (c) 依 FIFO（First key inserted is first removed）的順序原則將每個字母移除掉，並將每一步驟詳細畫出來。

5. [6 分] 試證：若 T 為一 complete m-ary tree，其 height 為 h 且具有 k 個 leaves，則 $k \le m^h$。

6. [12 分] 假設 text 爲〝*aaabaadaabaaa*〞，請用下列方法找尋 pattern〝*aabaaa*〞，並詳述搜尋步驟：
   (a) Brute Force。
   (b) Boyer-Moore Algorithm。
   (c) Knuth-Morris-Pratt Algorithm。

7. [5 分] 請找出下列網路之最大流量，並將每個步驟詳細畫出(圖中之 undirected edges 代表雙向流，且其兩個方向的流量相同)。



8. [10 分] 試證：假設 a、b、及 c 皆爲正整數，其中 $b \geq 2$，且 $f: \mathbf{Z}^+ \to \mathbf{R}$。如果

$$f(1) = c\ ;$$
$$f(n) = af(n/b) + c,\quad \text{for } n = b^k, k \geq 1,$$

則對於所有 n=1、b、b^2、b^3、…
   (a) 當 a=1 時，$f(n) = c(\log_b n + 1)$。
   (b) 當 $a \geq 2$ 時，$f(n) = \dfrac{c(an^{\log_b a} - 1)}{a-1}$。

9. [8 分] 設一 complete binary tree T 其 node 值爲 a、b、c、…、i、j 及 k。已知其按照 postorder 所得的序列爲 $d\ e\ b\ h\ i\ f\ j\ k\ g\ c\ a$，請分別考慮下列情形，將 T 之圖形畫出：
   (a) T 之 Height 爲 3。
   (b) T 之左子樹 Height 爲 3。

10. [5 分] 請將字串 "dogs do not spot hot pots or cats" 用 Huffman coding algorithm 來編碼，並將編碼的過程及結果寫出。

Problem 1 : Please find the minimal POS expression with a minimal number of variables for each of the following functions. (20%)

1). $Z_1 = \sum(0, 3, 5, 6)$

2). $Z_2 = \Pi(0, 2, 5, 7, 8, 10, 13, 15)$

Problem 2 : Please classify the following memories into three categories : (A) volatile, (B) non-volatile, (C) neither of the previous two categories. (10%)

SDRAM	SRAM
EEPROM	FLASH
OTP ROM	DRAM
MRAM	PROM
EPROM	FRAM

Problem 3 : MUXs are easily to be used to realize other Boolean functions. Referring to Figure 1, a 4-to-1 MUX is shown. Note that S_0 and S_1 are selection lines ($\overline{S_0}$ and $\overline{S_1}$ will be generated internally) while I_3, I_2, I_1, I_0 are the inputs. Z is the output. Assume that if $S_1S_0 = 11$, then $Z = I_3$; if $S_1S_0 = 10$, then $Z = I_2$, and so forth. You are ask to draw the schematic using the MUX to realize each of the following boolean expressions, respectively, given A and B are input signals. (10%)

$$Z_1 = \overline{A \cdot B} \quad Z_2 = \overline{A + B}$$
$$Z_3 = \overline{A} \quad Z_4 = A \oplus B$$

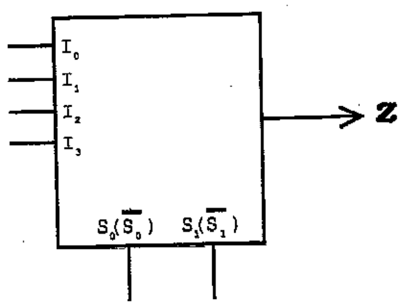


Figure 1: 4-to-1 MUX

Problem 4 : Try your best to use path sensitization to derive a test vector for the fault i/s-a-0 in the circuit of Figure 2. (10%)

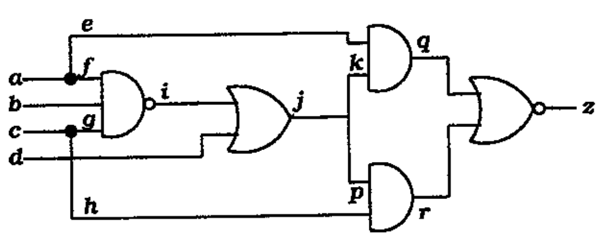


Figure 2: a circuit under test

Problem 5 : Derive a transition table and a state (flow) table for the circuit in Figure 3. (10%)

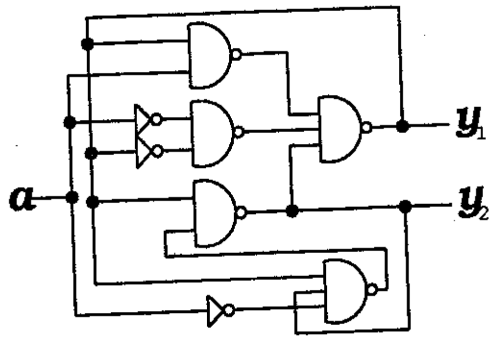


Figure 3: FSM of Problem 5

Problem 6 : Referring to the following state-transition table in Table 1, the input to the Finite State Machine (FSM) is a two-bit vector which will force the FSM to jump out the current state to the next state accordingly. Please find out a low-power state assignment of the FSM. You have to explain why your assignment can save power than a random assignment of the states. (10%)

current state	next state		
	input = 00	input = 01	input = 10
A	I	G	H
B	L	H	G
C	K	J	I
D	J	E	K
E	L	J	C
F	E	K	K
G	I	K	D
H	F	I	L
I	D	H	G
J	A	L	G
K	G	J	C
L	C	B	D

Table 1: FSM table of Problem 6

Problem 7 : Please draw a circuit containing only XOR gates and DFFs to realize the following encoder equation.

$$Z = f(x) = x^3 \bigoplus x \bigoplus 1$$

where x an input bit stream, and Z is an output bit stream. (10%)

Problem 8 : The microprocessor is the most powerful digital device in logic design, since it comprises many functional blocks, e.g., ALU, register file, etc. It also initiates transactions with external memory devices. The software code, i.e., the assembly language, is the tool to drive such a powerful device. Referring to the following 8-bit 8086/88 microprocessor codes, please write down which will NOT initiate memory transactions. (10%)

1). MOV [DI], AX

2). MOV CL, 0FH

3). SHL BYTE PTR [DI+BX], CL

4). SHR DX, CL

5). LAHF

Problem 9 : You are only allowed to use two-input byte-wide adders, shift registers, and DFFs to design a circuit to implement the following function.

$$Y = [2 \times X(t)] + [4 \times X(t - 1 \cdot T)] + [10 \times X(t - 2 \cot T)] + [6 \times X(t - 3 \cdot T)]$$

where $X(t)$ is the sampled value of an 8-bit input at time t, T is a delay which is identical to the delay of the DFFs. Please draw a block diagram of your design. (10%)

I. (25%) Computer System Performance

1. (10%) Define the following for a disk system:
   S = Seek time: average time to position head over track.
   R = Rotation speed of the disk, in revolutions per second.
   N = Capacity of a track, in bits.
   B = Number of bits per sector.
   T = Time to access a sector.
   Develop a formula for T as a function of other parameters.

2. (15%) Consider a computer system that contains an I/O module controlling a simple keyboard/printer teletype. The following registers are contained in the processor and connected directly to the system bus:
   ACC: Accumulator.
   INPR: Input Register, 8 bits.
   OUTR: Output register, 8 bits.
   FGI: Input flag, 1 bit.
   FGO: Output flag, 1 bit.
   IEN: Interrupt enable, 1 bit.
   Keystroke input from the teletype and printer output to the teletype are controlled by the I/O module. The teletype is able to encode an alpha numeric symbol to an 8-bit word and decode an 8-bit word to an alphanumeric symbol.
   (a) (10%) Describe how does the processor use the first five registers to control the I/O module, when keystroke input from the teletype and printer output to the teletype, respectively.
   (b) (5%) Describe how the function can be performed more efficiently by also employing IEN.

II. (25%) Instruction Set and Computer Arithmetic

1. (10%) Design a **variable-length opcode** and **operands** to allow all of the following to be encoded in a **40-bit** instruction.

   15 instructions with two **16**-bit addresses and one **4**-bit register number.
   1000 instructions with one **16**-bit address and one **4**-bit register number.
   50 instructions with no addresses or registers.
   <Note> You must show the max. and the min. number of bits in OP code and operands, respectively.

2. (15%) Computer Division Operation and Floating Point Representation
   (a) (5%) Express **-240** in **IBM's 32-bit** floating-point format (**Bias** = **64**), which uses a **7-bit** exponent with an implied base of **16**. Assume no normalization is used.
   (b) (10%) Divide **(-7)** by **(3)** in binary two's complement notation using **8-bit** word. The dividend **(-7)** is first loaded into **A** and **Q** registers and the divisor **(3)** is loaded into **M** register.
   <Note> You must show the steps of how to **shift** and **restore** the **8 bits** in **A** and **Q** registers.

國立中山大學九十一學年度碩士班招生考試試題

科目： 計算機結構（電機系）丙組） 共2頁 第2頁

III. (25%) Virtual Memory and Cache Memory

1. (10%) The following sequence of virtual page numbers is encountered in the course of execution on a computer with virtual memory: **3 4 2 6 4 7 1 3 2 6 3 5 1 2 3.**
   Assume that a LRU (Least Recently Used) page replacement policy is adopted. Calculate **Page Hit Ratio** (fraction of page references in which the page is in main memory) as a function of main memory page capacity N for $1 \le N \le 8$. Assume that the main memory is initially empty.

2. (15%) Consider a **512K** byte cache memory and a **256M** byte main memory hierarchy, in which

   Cache access time for every word = **10** ns.
   Main memory access time for every word = **100** ns.
   Cache block size = **64** bytes, Sector size = **16** blocks, Word size = **4** bytes.

   (a) (5%) Show the address format (how many bits in tag, set, and word) of a **4-way Set-Associate Mapping**.
   (b) (5%) Show the address format (how many bits in sector, block, and word) of a **Sector Mapping.**
   (c) (5%) Calculate Cache Miss Penalty (**CMP** in **ns**) by assuming that cache hit ratio = **0.9.**

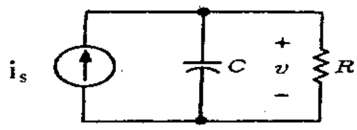
IV. (25%) Pipeline and Multiprocessor Architectures

1. (10%) For a **10-stage** linear pipeline, given the following data:
   N = the total number of tasks = 1,000
   t = clock cycle time = 1 ns
   D = Latch delay = 10 ns.
   H = the cost of each latch = 0.1 dollars.
   C = the total cost of all logical gates of stages = 100 dollars.
   T = the total time required to execute a non-pipeline sequential program = 10 us.
   (a) (5%) Calculate the optimal number of stages to maximize the PCR (Performance-Cost Ratio).
   (b) (5%) Calculate the Speedup, Efficiency, and Throughput of this linear pipeline.

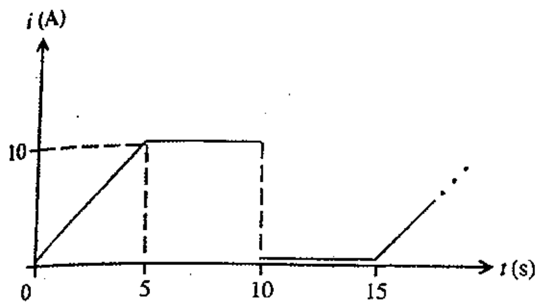
2. (15%) Multi-computers and Multiprocessors.
   (a) (5%) Calculate the end-to-end communication latency for a **10,000-byte** packet traversed through **1000** nodes (including source and destination nodes) using **store-and-forward**, assuming that the channel bandwidth is **10 Mbits/sec.**
   (b) (5%) Based on the same conditions, calculate the communication latency using **worm-hole routing** with the **10,000-byte** packet being divided into **1000** small flits.
   (c) (5%) Identify three major **differences** between message-passing multi-computers and shared-memory multiprocessors.

Solve the following problems (20% each). Write down the solutions explicitly according to the given order, and attach the derivations on the solution sheet.

(Problem 1) For capacitor C, Let Vc(0)=0, $i_s(t)=\delta(t)$ find Vc(t) = __(1)__



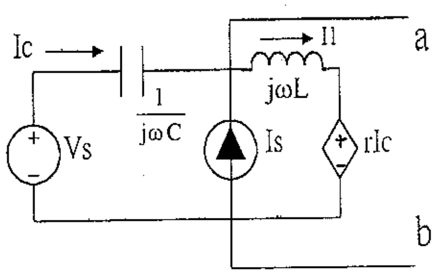
(Problem 2)



Current *i* is periodic as shown above. Determine the rms value (effective value) of current *i*. What average power will this current deliver to a 12-Ω resistance?

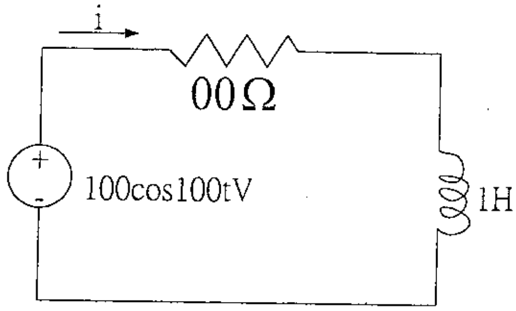
i = __(2)__ ; P_{av} = __(3)__

(Problem 3) For the following circuit, find the Thevenin and Norton Equivalent Circuit.



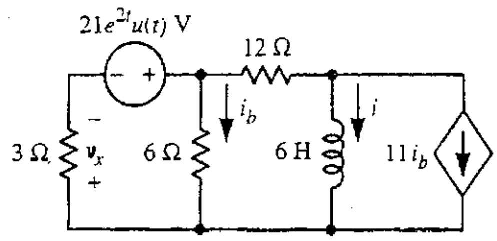
Vo = __(4)__ ; Rs = __(5)__ ; and draw the circuit __(6)__

(Problem 4) For the circuit with $V(t)$=100cos100t



(1) Find pf= (7)
(2) Change pf to 0.95Add a capacitor C = (8)

(Problem 5) In the following network, find current i and voltage v_x for t>0



i = (9) ; v_x = (10)

國立中山大學九十一學年度碩士班招生考試試題

科目：電力系統(電機系)(丁組)　　　　　共一頁第一頁

1. (a) For Fig.1, solve the voltage magnitude and phase angle of Bus 2 by Newton-Raphson method for two iterations. (Bus 1 is the slack bus and the initial value of V_2 is $1.0\angle 0^\circ$. (15%)

   (b) Based on the results in (a), calculate the line loss. (5%)

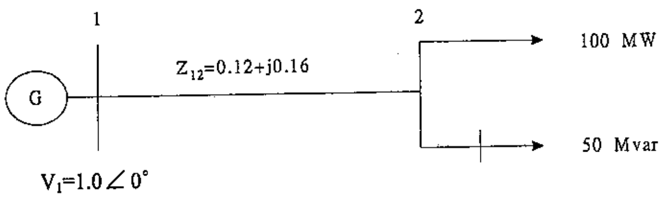


Fig. 1

2. The fuel-cost functions for three generator units in $/h are given by

$C_1=312.5+8.25P_1+0.005P_1^2$　　$50\le P_1\le 250MW$

$C_2=135+9.98P_2+0.006P_2^2$　　$5\le P_2\le 150MW$

$C_3=60+9.76P_3+0.006P_3^2$　　$15\le P_3\le 400MW$

Solve the optimal dispatch if the total load is 450MW. (15%)

3. For Fig.3, a fault occurs at Bus 3 with fault impedance of Z_f=j0.19. Calculate the fault current and the voltage magnitude of Bus 3 during fault. (All impedances in p.u.) (15%)

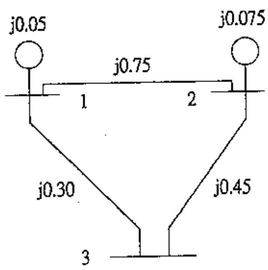


Fig. 3

4. (a) For the system in Fig.4, a generator with an inertia constant of 5.66MJ/MVA is connected to an infinite loss. All impedances are expressed on a common MVA base. The generator is delivering a real power of 0.77 p.u. to Bus 1. Solve the generator excitation voltage and obtain the swing equation. (15%)

   (b) When a bolted fault occurs at point f in Fig.4, Determine the critical clearing time by graph method. (10%)

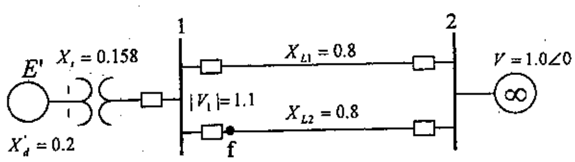


Fig. 4

5. A 208-V, 35-kVA, 0.85 PF leading, Y-connected, 60-Hz synchronous motor has a synchronous reactance of 2 Ω and a negligible armature resistance. It has a friction & windage loss of 1.2 kW and a core loss of 1.0 kW. If the motor terminal voltage magnitude is kept at its rated value, what are the motor line current (per-phase) $\mathbf{I}_a$ and internal generated voltage $\mathbf{E}_a$ when the motor shaft is supplying a 20-hp load at 0.85 PF leading? What will be the new operating current and power factor if the output of shaft is increased to 30-hp (with fixed field excitation)? (25%)

國立中山大學九十一學年度碩士班招生考試試題

科目：電磁學（電機系）（戊組）

1. Two conductors are immersed in a homogeneous lossy dielectric with dielectric constant ε and conductivity σ. A current will flow from the positive to the negative conductor and a current density field will be established in the medium. Prove that the product of the capacitance and the leakage resistance of this two-conductor system is a constant. (15%)

2. There is a spherical dielectric centered at the origin in free space. The relative dielectric constant is 10 and the radius of the sphere is 2. Inside this dielectric sphere there are three charges $+Q$, $-2Q$, $+3Q$ located at (1, 0, 0), (0, 1, 0), and (0, 0, 1), respectively. Find the outward flux of the electric field over the spherical surface at a distance $r = 10$. (15%)

3. There are two infinite plane current sheets located $\lambda/4$ apart in free space, each carrying a current of the form

$$\mathbf{J}_{s1} = \hat{\mathbf{a}}_x J_0 \cos\omega t \quad , \quad \text{at } z = 0$$
$$\mathbf{J}_{s2} = \hat{\mathbf{a}}_x J_0 \cos(\omega t + \alpha) \quad , \quad \text{at } z = \lambda/4$$

respectively. Find the ratio of the amplitude of the electric field in the region $z > \lambda/4$ to that in the region $z < 0$. (20%)

4. Derive the expressions of field components of the TM waves inside a rectangular waveguide. Start from solving for the longitudinal component from the Helmholtz equation, and then the rest of the field components from the longitudinal component. (20%)

5. Briefly answer the following questions. (5% each)
   (a) What is a uniform plane wave? Write down the mathematical form of a uniform plane wave propagating in the y-direction in a Cartesian coordinates.
   (b) Explain why single-conductor waveguide can not support TEM waves.
   (c) What is the essential difference between a transmission line circuit and an ordinary electric circuit?
   (d) What is a Smith chart? Where is the point representing a matched load on a Smith chart?
   (e) Does the magnetic field intensity due to a current distribution depend on the properties of the medium? Does the magnetic flux density?
   (f) What is the basic principle in applying the method images for solving the potential distribution due to electric charges?

國立中山大學九十一學年度碩士班招生考試試題

科目：線性代數（電機系）（乙組）　　　共 2 頁 第 1 頁

計分說明:本題計分將採用一種新創的"市場需求調整計分法"，即計分高低(價值與獲解率(市場需求率之倒數，愈易獲解，市場上需求愈低)成反比。所以考生除了要把握多數人會的題目外，還要著重於自己獨特的能力，即別人不容易會的問題。 在入學徵才錄取率不高的情況下，希望考生能發揮出自己卓越的特點。

此框可以不讀　你的得分($S_1', S_2', S_3' \ldots$)公式計算如下：令$R_1, R_2, R_3, \ldots$爲小題各自原始分數給分範圍，$R_1', R_2', R_3' \ldots$爲依市場需求調整後的分數給分範圍，$M_1, M_2, M_3, \ldots$爲小題各自的原始平均得分，$S_1, S_2, S_3, \ldots$ 爲小題各自原始得分：

則$S_1' = S_1 \times \frac{R_1'}{R_1}$　而$R_1', R_2', R_3', \ldots$的計算，在$M_1, M_2, M_3, \ldots$的各小題平均分數算出後

依據$R_1' : R_2' : R_3', \ldots = \frac{R_1}{M_1} : \frac{R_2}{M_2} : \frac{R_3}{M_3}, \ldots$得到，且$R_1' + R_2' + R_3', \ldots = 100$

注意$R_1, R_2, R_3, \ldots$的原始範圍可以任意設定，爲著讓考生易所依循，我仍可以做一次不必要的設定：$R_1 = R_2 = R_3, \ldots = \frac{100}{5}$。

1. Prove or disprove the following statements;

(1) The intersection of two vector spaces is a vector space.

(2) The union of two vector spaces is a vector space.

2. Let $AX = B$ be a system of m equations in n unknowns. Please give the necessary and sufficient condition for solution existing. (There is at least one solution)

3. Let T be a linear operator space V of dimension 3, and let x be a vector in V. If p denotes the smallest positive integer such that $(T\text{-}2I)^3(x) = 0$, and $(T\text{-}2I)^2(x)$, $(T\text{-}2I)(x)$, x are independent vectors,　then

(1)What is the matrix presentation of T by the ordered set $\{(T\text{-}2I)^2(x), (T\text{-}2I)(x), x\}$?

(2) Find the eigenvectors of T.

4. Provide **reasons** for those of which who are <u>**not**</u> inner product on the given vector spaces.

(a) $\langle x,y\rangle = xAy^*$ on C^2 where $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) $\langle x,y\rangle = xAy^*$ on C^2 where $A = \begin{pmatrix} 2 & i \\ -i & 1 \end{pmatrix}$

(c) $\langle A, B\rangle = tr(A + B)$ on M_{2*2} (R), where tr denotes the conventional trace operation

$tra(A) = \sum_i A_{ii}$.

(d) $\langle A, B\rangle = tr(A^* B)$ on M_{2*2} (R), where tr denotes the conventional trace operation

5. Reading Comprehension:

Theorem 1.10 (Replacement Theorem). Let V be a vector space having a basis β containing exactly n elements. Let S={ $y_1,\dots,y_m$ } be a linearly independent subset of V containing exactly m elements, where m$\le$n. Then there exists a subset S_1 of β containing exactly n-m elements such that $S\cup S_1$, generates V.

Proof. The proof will be by induction on m. The induction begins with m = 0; for in this case S=ϕ, so $S_1=\beta$ clearly satisfies the conclusion of the theorem.

Now assume that the theorem is true for some m, where m < n. We will prove that the theorem is true for m+1. Let S={ $y_1,\dots,y_m,y_{m+1}$ } be a linearly independent subset of V containing exactly m+1 elements. Since { $y_1,\dots,y_m$ } is linearly independent by the corollary to Theorem 1.6, we may apply the inductive hypothesis to conclude that there exists a subset { $x_1,\dots,x_{n-m}$ } of β such that { $y_1,\dots,y_m$ } $\cup$ { $x_1,\dots,x_{n-m}$ } generates V. Thus there exist scalars $a_1,\dots,a_m,b_1,b_2,\dots,b_{n-m}$ such that

$$y_{m+1}=a_1y_1+\cdots+a_my_m+b_1x_1+b_2x_2+\cdots+b_{n-m}x_{n-m} \qquad (9)$$

Observe that some b_i say b_1, is nonzero, for otherwise (9) would imply a contradiction to the assumption that **(a)**. Solving (9) for x_1, gives

$$x_1=(-b_1^{-1}a_1)y_1+\cdots+(-b_1^{-1}a_m)y_m-(-b_1^{-1})y_{m+1}+(-b_1^{-1}b_2)x_2+\cdots+(-b_1^{-1}b_{n-m})x_{n-m}$$

Hence $x_1\in$ span({ $y_1,\dots,y_m,y_{m+1},x_2,\dots,x_{n-m}$ }). But since $y_1,\dots,y_m,x_2,\dots,x_{n-m}$ are clearly elements of span({ $y_1,\dots,y_m,y_{m+1},x_2,\dots,x_{n-m}$ }), it follows that

$$\{y_1,\dots,y_m,x_1,x_2,\dots,x_{n-m}\}\subseteq \text{span}(\{y_1,\dots,y_m,y_{m+1},x_2,\dots,x_{n-m}\}).$$

Thus Theorem 1.5 implies that

$$\text{span}(\{y_1,\dots,y_m,y_{m+1},x_2,\dots,x_{n-m}\})=V.$$

So the choice of S_1= **(b)** proves that the theorem is true for m + 1.

This completes the proof.

(a) What is the assumption in **(a)**?

(b) What is the Set S_1 in **(b)**?

(c) What should be Theorem 1.5 about?

1. For the uniform random variable defined by

$$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a\le x\le b\\ 0 & ,otherwise\end{cases}.$$

Where a and b are arbitrary real values. (a) Find $P_r\{0.9a+0.1b < X \leqq 0.7a+0.3b\}$ (10%)

(b) $P_r\{(a+b)/2 < X \leqq b\}$ (10%) (**Note**: $P_r\{A\}$ denotes the probability of event A).

2. Assume that X is a random variable and **Y** is another random variable proportional to the power of the random variable X, e.g., $y=g(x)=x^2$, for $x\geqq 0$, and is zero otherwise.

   (a) Given $f_X(x)$ of random variable X, find the probability density function of random variable Y, $f_Y(y)$. (10%)

   (b) Find $f_Y(y)$ if random variable X is Gaussian distribution (5%).

3. The time that it takes to serve a customer at the cash register in a mini-market is a random variable having an *exponential distribution* with parameter λ, i.e.,

$$f_T(t)=\lambda\exp[-\lambda t],\qquad t\ge 0$$

Supposed T_1 and T_2 are service times for two different customers, assumed independent of each other. Consider the total service time $T_a = T_1 + T_2$ for the two customers, also a statistic.

(a) Find the cumulative distribution function of T_a, F_{Ta}(10%)

(b) The probability density function of T_a, $f_{Ta}(t)$ (5%).

4. Let the random variables X and Y be independent and Gaussian, and let each have a mean of zero and a variance of σ^2. If a new random variable Z is defined by

$$Z=a\frac{X}{Y}$$

   (a) Find the conditional probability density function of Z given Y. (8%)

   (b) What is the probability density function of Z. (7%),

5. The power (in milliwatts) returned to a radar from a certain class of aircraft has the probability density function

$$f_P(p)=\frac{1}{10}e^{-p/10}u(p)$$

Suppose a given aircraft belongs to this class but is known to not produce a power larger than 15 mW. Where $u(p)$ is denoted as the *unit step* function of p.

(a) Find the probability density function of P conditional on P $\leqq$ 15 mW. (7%)

(b) Find the conditional mean value of P. (8%)

6. Random variables X and Y have respective density functions

$$f_X(x)=\frac{1}{a}[u(x)-u(x-a)]$$

$$f_Y(y)=bu(y)e^{-by}$$

For a>0 and b>0. Where $u(x)$ is denoted as the *unit step* function of x. (a) Find and sketch the density function of W=X+Y if X and Y are statistically independent. (10%) (b) What is the characteristic function of W. (10%)

國立中山大學九十一學年度碩士班招生考試試題

科目：通訊理論（電機系）（乙組）

Communications: 10 points for each problem

1. What is "noise figure" of a system? Describe its importance. Derive the overall noise figure for a three-stage cascaded system with stage gain G1, G2, G3, and corresponding noise figure F1, F2 and F3.

2. Differential pulse code modulation DPCM can be used to reduce the data rate of a communication system if the data samples are highly correlated. If the first difference of a data stream x(r) is d(r)=x(r)-x(r-1), r=1,2,3 ⋯. For a zero-mean stationary random process, derive the condition for adjacent sample correlation coefficient such that DPCM is NOT better than PCM.

3. Why do we use "compander"? Describe its importance. How can we design mathematical equations to "compress" and "expand" the signals in mobile communication systems.

4. Why and how do we use envelope detection for "commercial" AM system (DSB/LC) instead of other techniques? What are the other techniques?

5. Why and how do we use preemphasis and deemphasis techniques for FM SNR improvement?

6. What roles are the "Wiener filter" and "matched filter" playing in communication systems? Explain their importance and derive mathematical equations for their implementations.

7. What is "Gram-Schmidt Procedure"? Explain its basic idea and derive the mathematical equations.

8. What are the differences among Bayesian decision rule, maximum a posteriori decision rule and maximum likelihood decision rule? Derive the optimal threshold for detection of Gaussian random signal corrupted by independent additive white Gaussian random noise.

9. Derive the probability of error for binary orthogonal signals. How can the union bound be used to calculate the probability of error for M-ary orthogonal signals?

10. What are the differences among CDMA, FDMA and TDMA? Design a circuit to generate m-sequence of length 31. Why do we use Gold codes and how to generate them.