國立中山大學97學年度碩士班招生考試試題

科目：電子學【電機系碩士班甲、乙、戊組；通訊碩乙組；通訊領域聯合招生甲組】

1. (10%) Design a non-inverting amplifier with a gain of 1.5 V/V using three 100 kΩ resistors.

2. (10%) A 1-mA diode having a 0.1 V/decade characteristic operates from a constant-current supply with V_D = 0.8 V. If it is shunted by two more identical diodes, what does the voltage drop become?

3. (10%) For the circuit shown in Fig. 1, find I_C and V_{CE} for V_{BE} = 0.7 V and β = 50.

4. (10%) For the FET circuit shown in Fig. 2, I_{DSS}=4 mA and V_P = −2 V. Find I_D and V_o.

5. (10%) For the circuit shown in Fig. 3, $R_1 = R_2$ = 10 kΩ and $C_1 = C_2$ = 100pF. Find the upper 3-dB frequency exactly.

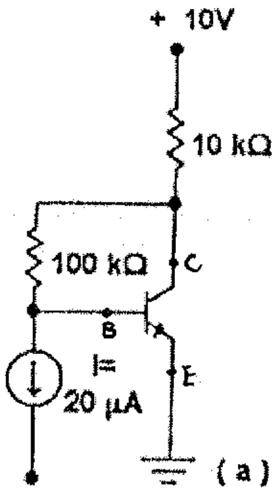

Figure 1

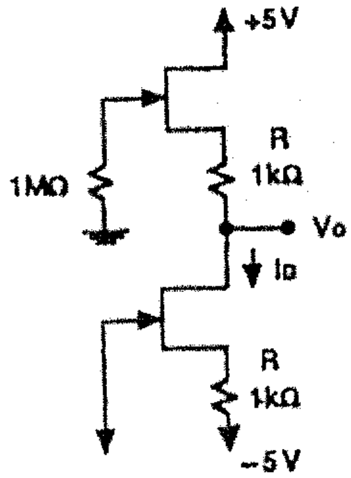

Figure 2

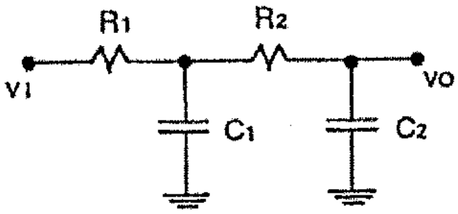

Figure 3

6. (20%) The feedback amplifier in Figure 4 has I = 1mA and V_{GS} = 0.8 V. The MOSFET has V_t = 0.6V and V_A = 30 V. For R_s = 10 kΩ, and R_1 = 1 MΩ, and R_2 = 4.7 MΩ, find (a) the feed-back configuration, (b) the voltage gain v_o / v_s, (c) the input resistance R_{in}, and (d) the output resistance R_{out}.

7. (10%) Using a simple (r_π, g_m) model for each of the two transistors Q_{18} and Q_{19} in Figure 5, find the small-signal resistance between A and A' assuming I_{C18} = 165 μA and I_{C19} = 16μA.

8. (20%) Figure 6 shows the circuit for determining the op-amp output resistance when v_o is positive and Q_{14} is conducting most of the current. Using the resistance of the Q_{18}-Q_{19} network calculated in Figure 5 and neglecting the large output resistance of Q_{13A}, find R_{out} when Q_{14} is sourcing an output current of 5 mA.

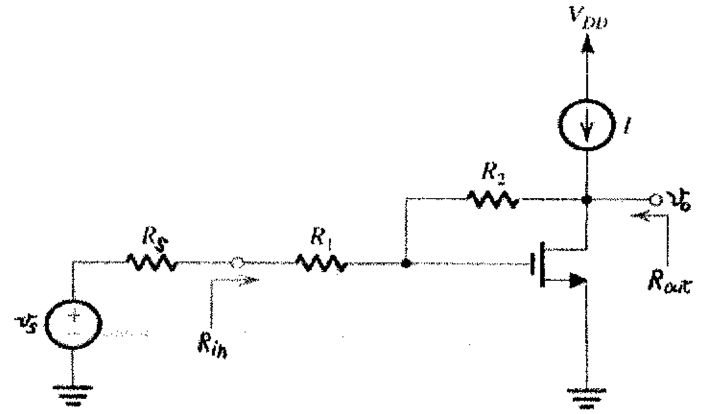

Figure 4

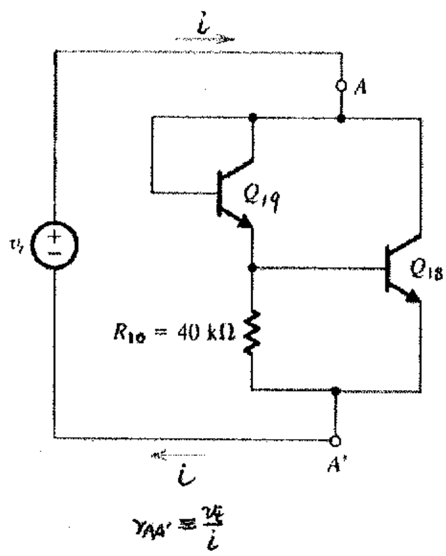

Figure 5

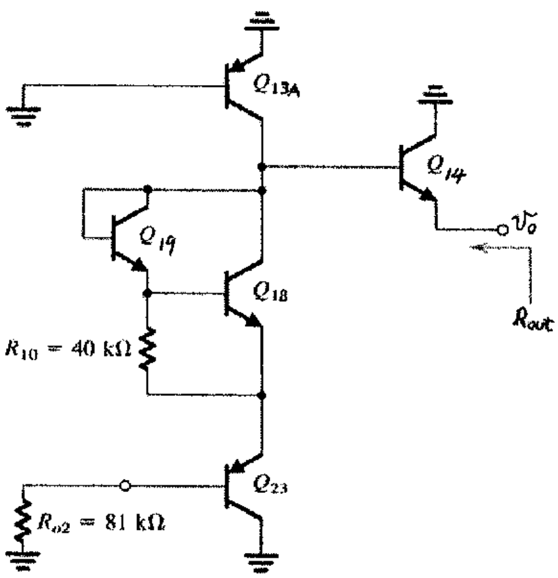

Figure 6

1. (10%) Solve the initial value problem (IVP) for the following ODE

$$y'' + y = 5x + 8\sin x, \quad y(\pi) = 0,\ y'(\pi) = 2.$$

2. (15%) Find the solution of the initial value problem

$$y'' + 2y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

3. (5%) Find the Laplace transform of (a) $\mathcal{L}\{e^{-5t}\}$ (b) $\mathcal{L}\{\sin 3t\}$

4. (15%) Solve the initial value problem by the Laplace transform

$$\begin{cases} y_1' + 2y_2' = 1 \\ 3y_1' + y_2' + y_2 = t \end{cases} \quad y_1(0) = 0,\ y_2(0) = 0$$

5. (15%) Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 2, & 0 \le x < \pi \end{cases}$ in a Fourier series.

6. (13%) Evaluate the following integral

$$\oint_C \frac{dz}{\sinh\ (2z)},$$

where z is a complex variable and C denotes the circle $|z| = 2$ described in the positive sense.

7. (15%) The set

$$S = \left\{\frac{1}{\sqrt{2}},\ \cos\ x,\ \cos\ 2x,\ \cos\ 3x,\ \cos\ 4x\right\}$$

is an orthonormal set of vectors in $C[-\pi,\ \pi]$ with inner product defined as

$$\langle f,\ g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\ dx,$$

where $C[-\pi,\ \pi]$ is the set of all functions f that are continuous on $[-\pi,\ \pi]$. Suppose that the function $\sin^4 x$ can be written in a linear combination of elements of S as

$$\sin^4 x = \frac{3\sqrt{2}}{8}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}(\cos\ 2x) + \frac{1}{8}(\cos\ 4x).$$

Use the above equation and orthogonal basis property (but do not compute antiderivatives, otherwise you will get zero credit), find the values of the following integrals:

(i) $\int_{-\pi}^{\pi} \sin^4 x\ dx$ (ii) $\int_{-\pi}^{\pi} \sin^4 x \cos(3x)\ dx$ (iii) $\int_{-\pi}^{\pi} \sin^4 x \cos(4x)\ dx$

8. (12%) Let P_4 be the set of all polynomials of degree less than 4. In P_4 the inner product is defined by

$$\langle p,\ q\rangle = \sum_{i=1}^{4} p(x_i)q(x_i),$$

where $x_i = (i-2)/2$ for $i = 1, \cdots, 4$. Its norm is defined by

$$\|p\| = \sqrt{\langle p,\ q\rangle} = \left\{\sum_{i=1}^{4}[p(x_i)]^2\right\}^{1/2}.$$

Compute (a) $\|x^2\|$, (b) the distance between x and x^2.

1 Consider a silicon p^+-n junction diode. The avalanche breakdown happens when the maximum junction electric field equals to the critical field E_{crit}. Calculate the breakdown voltage for $N_D = 10^{16}$ cm^{-3}. (For silicon: $\varepsilon_{Si} = 11.8\ \varepsilon_0$, $\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm, $E_{crit} = 3.5 \times 10^5$ V/cm, also $q = 1.6 \times 10^{-19}$ C) (20%)

2 Consider a abrupt p^+-n junction. Derive the expression of the capacitance with respect to the reverse biased voltage. (20%)

3 Derive the I-V expression of a abrupt p^+-n junction diode. (20%)

4 Describe the physical principle of the C-V curve of a MOS capacitor. (20%)

5 Describe the physical principle of a semiconductor laser. (20%)

1. (15%) Find the solution of the initial value problem

$$y'' + 2y' - 3y = 0,\quad y(0) = 1,\quad y'(0) = 1.$$

2. (5%) Find the Laplace transform of (a) $\mathcal{L}\{e^{-5t}\}$ (b) $\mathcal{L}\{\sin 3t\}$

3. (15%) Solve the initial value problem by the Laplace transform

$$\begin{cases} y_1' + 2y_2' = 1 \\ 3y_1' + y_2' + y_2 = t \end{cases} \quad y_1(0) = 0,\ y_2(0) = 0$$

4. (15%) Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 2, & 0 \le x < \pi \end{cases}$ in a Fourier series.

5. (10%) Find an orthogonal matrix P that diagonalizes

$$A = \begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & 0 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$

6. (13%) Find the best quadratic polynomial to fit the data $p(-1) = 0$, $p(0) = 1$, $p(1) = 2$, $p(2) = 4$.

7. (15%) The set

$$S = \left\{ \frac{1}{\sqrt{2}},\ \cos x,\ \cos 2x,\ \cos 3x,\ \cos 4x \right\}$$

is an orthonormal set of vectors in $C[-\pi, \pi]$ with inner product defined as

$$\langle f, g \rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\,dx,$$

where $C[-\pi, \pi]$ is the set of all functions f that are continuous on $[-\pi, \pi]$. Suppose that the function $\sin^4 x$ can be written in a linear combination of elements of S as

$$\sin^4 x = \frac{3\sqrt{2}}{8}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}(\cos 2x) + \frac{1}{8}(\cos 4x).$$

Use the above equation and orthogonal basis property (but do not compute antiderivatives, otherwise you will get zero credit), find the values of the following integrals:

(i) $\int_{-\pi}^{\pi} \sin^4 x\,dx$ (ii) $\int_{-\pi}^{\pi} \sin^4 x \cos(3x)\,dx$ (iii) $\int_{-\pi}^{\pi} \sin^4 x \cos(4x)\,dx$

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$$\langle p, q \rangle = \sum_{i=1}^{4} p(x_i)q(x_i),$$

where $x_i = (i-2)/2$ for $i = 1, \cdots, 4$. Its norm is defined by

$$\| p \| = \sqrt{\langle p, q \rangle} = \left\{ \sum_{i=1}^{4} [p(x_i)]^2 \right\}^{1/2}.$$

Compute (a) $\| x^2 \|$, (b) the distance between x and x^2.

1. Figure 2 shows the block diagram of the circuit in Fig. 1.
   (a) (10%) Determine the transfer functions C and P.
   (b) (10%) Sketch the Bode diagram of P
   (c) (5%) Determine the voltage gain k such that the feedback system has a phase margin of roughly 45 degrees.
   (d) (5%) Given k=100 and a 10-volt dc input r. Find the steady-state value of x.

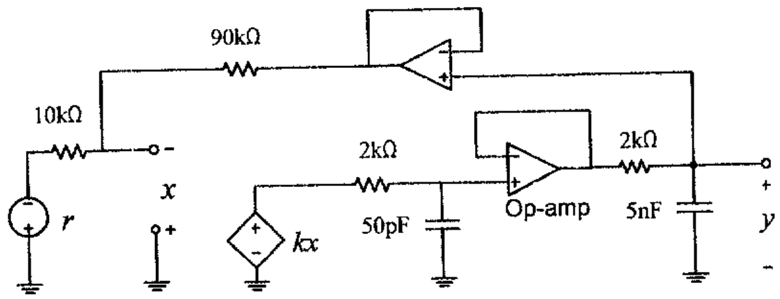


Fig.1

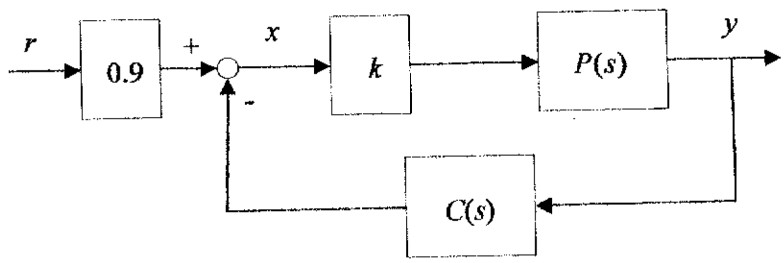


Fig.2

2. (15%) Given $C(s)=\dfrac{s+1}{s(s-1)}$ and $P(s)=\dfrac{s-1}{s+1}$ in Fig. 3. Is the feedback system capable of tracking a unit step input with zero steady state error? Why?

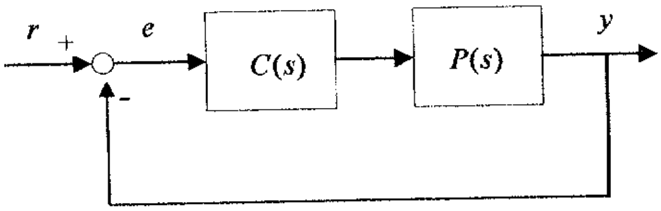


Fig.3

3. (20%) Given $P(s)=(s+1)/(s+2)$ in Fig. 3. Design a feedback controller C such that the plant's output asymptotically tracks $r = \sin(5t)$ without steady-state error.

4. (20%) For the circuit of Fig.4, please find its state space representation with the state variables being x_1 and x_2, the input being u, and the output y. Also find its characteristic roots.

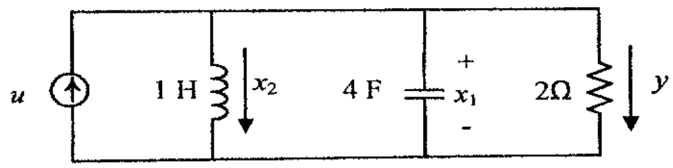


Fig. 4

5.(15%) Suppose a linear system has an impulse response $h(t)$. Prove that if

$$\int_0^\infty |h(t)|\,dt < \infty$$

then the system is BIBO stable.

1. [10] Please work out the following problems:
   a. [5] We define $f(n) = O(g(n))$ if and only if there exist positive constants c and n_0 such that $f(n) \leq cg(n)$ for all n, $n \geq n_0$. Using this definition, show that $10n^2 - 4n + 20 = O(n^4)$.
   b. [5] We define $f(n) = \Theta(g(n))$ if and only if there exist positive constants c_1, c_2, and n_0 such that $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all n, $n \geq n_0$. Using this definition, show that $6 * 3^n + n^2 = \Theta(3^n)$.
2. [10] Suppose we have the following recursive function for making a sum of n integer numbers stored in an array:

```
Int RecursiveSum(int element[], int n)
{
    If (n)
        return (RecursiveSum(element, n-1) + element[n-1]);
    return 0;
}
```

   a. [5] Let the time complexity of this function is t(n). Please derive a recursive formula for t(n) indicating the recurrence relation between t(n) and t(n-1).
   b. [5] Please solve the recurrence and describe the time complexity in Θ notation.
3. [10] Suppose each word occupies one address in a computer.
   a. [5] What is the address of the element $A[i_1][i_2]...[i_n]$ in an array declared as $A[u_1][u_2]...[u_n]$? Assume that a column-major representation of the array is used, with one word per element and s being the address of A[0][0]...[0].
   b. [5] What is the address of the element A[2][3][4] in an array declared as A[10][20][30]? Assume s is 500 in this case.
4. [10] Please provide a definition of the abstract data type for Set. The minimal operations should include: Create, Insert, Remove, IsIn, IsEmpty, Union, and Intersection.
5. [10] A full d-ary tree is a d-ary tree in which every internal node has d children and all the leaf nodes have the same depth. Note that a one-node d-ary has a depth of 1.
   a. [5] What is the total number of nodes in a full d-ary tree with a depth of k?
   b. [3] Suppose we number the nodes in a full d-ary tree starting with the root on level 1, continuing with the nodes on level 2, and so on. Nodes on any level are numbered from left to right. Let the first node be numbered 1, the second be numbered 2, and so on. For an internal node numbered with t, what are the numbers assigned to its d children respectively?
   c. [2] A d-ary tree with n nodes and depth k is complete if and only if its nodes

correspond to the nodes numbered from 1 to n in the full d-ary tree of depth k. What number is assigned to the last internal node?

6. [10] Please work out the following problems:
   a. [5] Suppose we have a binary tree each of its nodes containing a data field for storing a lower-case character. We know that the preorder sequence of the binary tree is ikjghbfcaed, and the inorder sequence is jkhbgicafde. Please construct and draw this binary tree.
   b. [5] We define a traversal rlc-order as follows:

```
void rlc-order(treePointer ptr)
{
    If(ptr)
        rlc-order(ptr -> rightChild);
        rlc-order(ptr -> leftChild);
        printf("%c", ptr -> data);
}
```

   Please write out the rlc-order sequence for the binary tree of Problem 6(a).

7. [10] Please prove the following statements:
   a. [5] The sum of the degrees of the vertices of an undirected graph is twice the number of edges.
   b. [5] For an undirected graph G with n vertices, ``G is a tree'' is equivalent to ``G contains no cycles and has $n-1$ edges''.

8. [10] Suppose we have a mechanism for searching, inserting, and deleting keys. Assume that the keys are integer and in the range [0,m-1] and that n is the number of insertions to be made. We use two arrays, a[n] and b[m] where a[i], $0 \le i < n$, stores the key of the (i+1)th insertion, and if k is the key of the (i+1)th insertion, then b[k]=i.
   a. [5] Suppose m=10 and we do 5 insertions of the following five integers 5, 1, 9, 4, 0, in order. Assuming all the elements in the arrays a and b are 0 initially. Please show the contents of the arrays a and b after the insertions.
   b. [5] What will the time complexity be if we want to search a key? Why?

9. [10] Suppose we have a sequence of 15 integer keys: 80, 70, 10, 60, 30, 95, 20, 65, 40, 55, 72, 58, 62, 52, 90.
   a. [5] Please construct a B-tree of order 3, by inserting the keys in the specified order. Draw the resulting B-tree.
   b. [5] Please delete 60 from the tree. Draw the resulting B-tree. Assume that a non-leaf element is replaced by the element with the largest key smaller than itself. Also, the right sibling is considered before the left sibling.

10. [10] The 2-d tree is a modification to the binary search tree (BST) that allows for

efficient processing of 2-dimensional keys. A node N, with key being (N_x,N_y), at an odd level, e.g., level 1, level 3, …, would have in its left subtree only nodes whose x-coordinate values are less than N_x. The right subtree would contain nodes whose x-coordinate values are greater than N_x. A node M, with key being (M_x,M_y), at an even level, e.g., level 2, level 4, …, would have in its left subtree only nodes whose y-coordinate values are less than M_y, and in its right subtree only nodes whose y-coordinate values are greater than M_y.

a. [5] To insert a new key into a 2-d tree, we search from the root (level 1) until a NULL pointer is found, indicating the proper place to insert the new node. Given the following sequence of 2-d keys: (30,60), (50,20), (20,80), (15,70), (40,45), (70,10), (55,85), (80,90), (75,25), (35,35), (45,95), (65,15), please construct a 2-d tree, starting from the null 2-d tree. Draw the resulting 2-d tree.

b. [5] To delete a node N from a 2-d tree, we replace the key stored in N by the key in N's right subtree with the least value of the proper coordinate. The substitution process is iterated until done. Please delete (55,85) from the 2-d tree obtained before. Draw the resulting 2-d tree.

（需將推導過程敘述清楚）

1. (25%)For 6 persons P = {John, Bob, Tom, Mary, Jane, Terry},
specified attributes of these persons with 6 vectors (person's name, person's age, person's favorite):
(John, 20, basket ball), (Bob, 15, volley ball), (Tom, 25, chess), (Mary, 18, volley ball), (Jane, 22, Music), (Terry, 15, Chess)
where John, Bob, and Tom are males, Mary, Jane, and Terry are females,
given rules:
(Rule 1) Any person who is less than or equal to 20 years old and favors playing a kind of ball, likes a person who favors playing chess.
(Rule 2) Any male person likes any female person who favors playing a kind of ball.
(Rule 3) Any person likes any person who is younger than he/she.
(a) (10%) Represent the like-relationship binary relation among these 6 persons in a relation matrix R.
(b) (10%) Derive the transitive matrix R^*.
(c) (5%) Find the set Q of persons q that are liked transitively by all persons in P.
$Q = \{q \mid q \in P, \forall p \in P, \exists \text{ path}_{like}(p,q)\}$

2. (10%) In order to organize a team of 5 persons to attend a programming contest, we can select team members from students from 4 departments
- Dept. of Electrical Eng. has 100 students to be selected: 1 selected EE student has weight 2
- Dept. of Computer Science has 50 students to be selected: 1 selected CS student has weight 2
- Dept of Mechanical Eng. has 80 students to be selected: 1 selected ME student has weight 1
- Dept. of Chemical Eng. has 40 students to be selected: 1 selected CE student has weight 1

What is the probability that the team of 5 persons should have total weight being 8?
(Write the formula of the result is ok as the answer.)

3. (10%) Given the following tautology:
$(A \wedge B) \vee ((B \vee \neg C) \rightarrow (C \wedge \neg A))$
where $\wedge$, $\vee$, and $\neg$ represent logical AND, logical OR, and logical NOT, respectively.
Decide and derive the following formula's truth value as one of the set element in {true, false, not known}.
(a) (5%) $A \wedge C$
(b) (5%) $\neg A \wedge (\neg B \vee C)$

4. (10%) In complete binary tree of depth d where all nodes are distinct nodes as an undirected graph, how many paths exists from any node at level i to any node at level j such that the path contains the root vertex?

5. (15%) Write a proof for the following statement:
Given an undirected tree graph, when we add an edge to the graph, the graph then have a Hamiltonian cycle. Prove that this tree graph is a chain graph.
(A chain graph G(V,E) has $V = \{v_1, v_2, \ldots, v_n\}$, $E = \{(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n)\}$)

6. (15%) Prove that in an undirected complete graph K_n of n distinct vertices, there are (n-1)! number of directed cycles of n vertices.

7. (15%) Given a directed graph G(V, E) where V contains all its vertices, and E contains all its directed edges, given a starting vertex V_s, write an recursive algorithm to output all vertices reacheable from V_s exactly once for such vertices.
(You can use a function target_vertex_set(V) that returns a list of vertices V_j's each of which is connected from V with an edge (V, V_j).)

[Problem 1] By Boolean algebra to determine and prove whether or not the following expressions are valid, i.e. whether the left- and right-hand sides represent same function. (10%)

a. $\overline{x\bar{y}(x+z)} + \bar{x}y\overline{(x+y+z)} = \bar{x}+y$

b. $\bar{x}_1\bar{x}_2\bar{x}_3 + \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3 = \bar{x}_3 + x_1\bar{x}_2$

[Problem 2] Design a circuit that implements the plus-six function as Figure P2 using the basic gates such as NOT, AND, OR, NAND, and NOR. (10%)

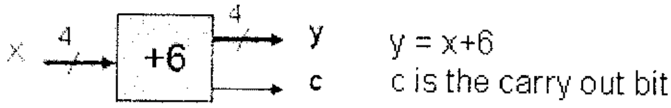


Figure P2. the plus-six function

[Problem 3] Design a circuit that implements the negative-edge-triggered D flip-flop with Clear and Preset using the basic gates such as NOT, AND, OR, NAND, and NOR. (20%)

[Problem 4] Design a circuit that has an input w and an output z. The machine is a sequence detector that produces $z = 1$ when the previous two values of w were 01 or 10; otherwise $z = 0$; (10%)

[Problem 5] Design a four-bit carry-lookahead adder using the basic gates such as NOT, AND, OR, NAND, NOR and XOR. (10%) By the four-bit carry-lookahead adder module (named CLA4), design a 16-bit carry-lookahead adder (5%). Let the basic gates take the same delay time as 1d. Determine the delay time of the critical path of the16-bit carry-lookahead adder. (5%)

[Problem 6]

(a) Write the code for a negative-edge-triggered D flip-flop with synchronous reset in VHDL or Verilog HDL. (5%)

(b) In VHDL or Verilog HDL, write the code for the traffic light controller, whose lighting sequences are as Figure P6. After the red lamp lights 3 cycles, the green lamp lights 2 cycles. After the green lamp lights, the yellow lamp lights 1 cycle. (15%)

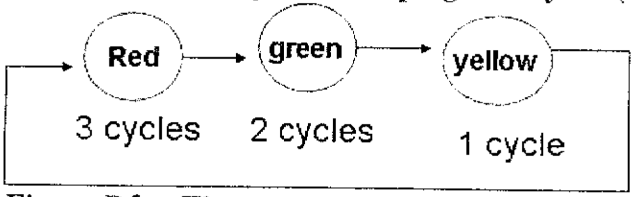


Figure P6. The sequences of the traffic light

[Problem 7] Derive an expression for the CMOS circuit in Figure P7. (10 %)

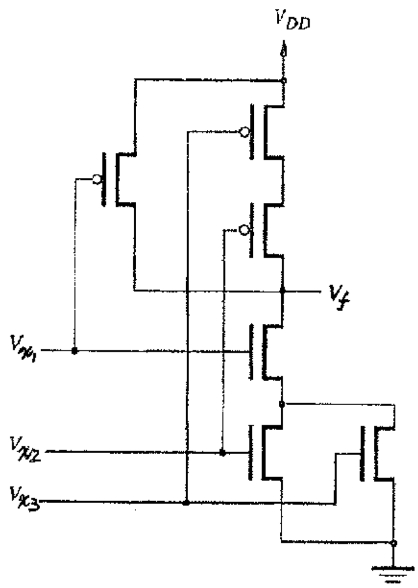


Figure P7. A three-input CMOS circuit

I. (20%) Identify **two** differences between the following computer terminology pairs.

(1) Interrupt-Driven I/O vs DMA

(2) SDRAM vs SRAM

(3) Write-Through Cache vs Copy-Back Cache

(4) Multiprogramming OS vs Multiprocessing OS

(5) Parallelism vs Pipelining

II. (20%) A 2.0 GHz microprocessor runs a program of 1000 assembly instructions. Given the following assumptions:

(i) all the instructions are 32-bit long and they all use immediate addressing mode.

(ii) one memory location can accommodate one 32-bit instruction.

(iii) an address or data bus cycle will take 2 CPU cycles.

(iv) address bus and data bus are both 16-bit wide.

(v) after an instruction is fetched from memory to instruction register, it requires 1 cycle for instruction decoding, 2 cycles for instruction execution, and 1 cycle for storing the result to register.

(vi) no instruction pipelining is allowed.

(1) Compute the number of CPU cycles required for executing the 1000 instructions.

(2) Compute CPI (Cycles per Instruction).

(3) Compute MIPS (Million Instructions per Second).

(4) Compute CPU throughput (Programs per Second).

III. (20%) A superpipelined superscalar processor of degree (**m**, **n**). This machine executes **m** instructions every cycle with a pipeline cycle **1/n** of the base cycle. Assume there are **N** independent instructions and **K** pipeline stages.

(1) Calculate T(1, 1), the time required for a base scalar machine (i.e., **m** = **n** = 1).

(2) Calculate the speedup factor, S(**m**, 1), of an **m**-issue superscalar machine (i.e., **n** = 1) over the base scalar machine.

(3) Calculate the speedup factor, S(1, **n**), of a superpipelined machine (i.e., **m** = 1) over the base scalar machine.

(4) Calculate the speedup factor, S(**m**, **n**), of a superpipelined superscaler machine over the base scalar machine.

IV. (20%) Write down the 32-bit IEEE 754 floating point representation of the following decimal numbers (Note that there are 1-bit sign, 8-bit biased exponent, 23-bit fraction, and bias = 127).

(1) 240

(2) $-55\frac{23}{64}$

V. (20%) A bus system consists of a multiplexed **32 bits** for transferring both address and data. There are **1 G** memory locations and **4** I/O modules to be referenced by CPU. A centralized arbitration scheme (<u>**NO**</u> hidden arbitration) with a clock rate of **40** MHz is used. Assuming that

(i) Memory latency time (the time required to latch the requested data on the bus) = **40 nsec**, and

(ii) Memory word length = **4 bytes**.

(iii) CPU always requests data in a block of **4 words**, and it takes **two bus cycles** for bus arbitration and **one bus cycle** for transferring address to main memory.

(1) Calculate the effective bus bandwidth (in Mbytes/sec), if CPU accesses the **4 words** at the same time in a Block Data Transfer Mode.

(2) Calculate the effective bus bandwidth (in Mbytes/sec), if CPU accesses the **4 words** one by one separately in a Read (Multiplexed) Operation Mode.

1. (a) (10%) Find the transfer function from input u to output y in the circuit of Fig. 1, under the assumption of ideal op-amps.
   (b) (5%) What kind of filters does it belong to (lowpass, bandpass, or highpass)?
   (c) (10%) Determine the steady-state output $y(t)$, given an input $u(t)=\sin(25t)$.

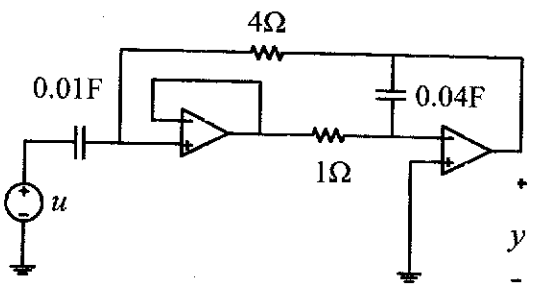


Fig. 1

2. (20%) Consider the circuit in Fig. 2, where the motor is modeled as a 4H inductor in series with a 4Ω resistor. Determine the capacitance value C so that the current following through the motor has the maximum amplitude. Find the average power consumed by the motor in this case.

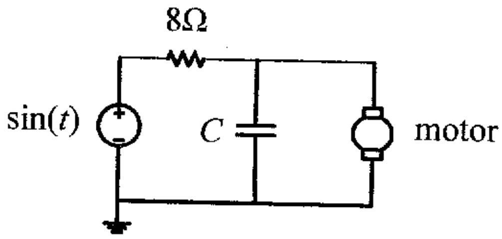


Fig. 2

3. (20%) For filtering, a unity-coupled transformer combined with a parallel capacitor is inserted between a 10kΩ resistive load and an ac voltage source with a 100Ω output resistance, as shown in Fig. 3. Choose the values of L_1 and L_2 to maximize the power delivered to the load when V_s is 10^7 rad/sec.

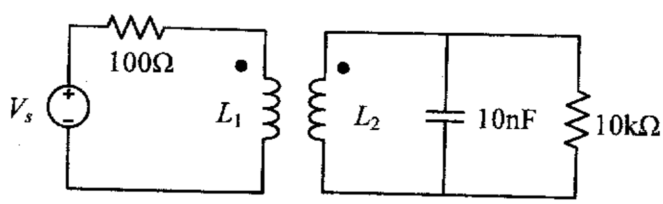


Fig. 3

4. (10%) The op-amp in Fig. 4-1 is ideal except that its output cannot swing beyond the supply voltages (that is, $-5 \le y \le 5$). Assume zero initial condition. Draw the output waveform $y(t)$ of the circuit in response to $u(t)$ given in Fig.4-2.

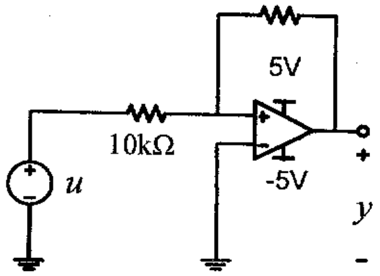


Fig.4-1

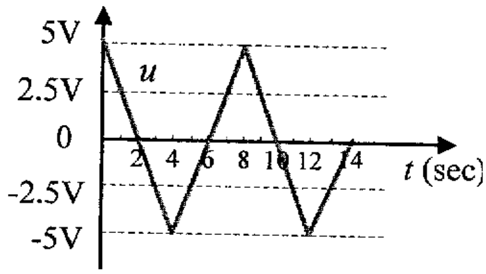


Fig. 4-2

5. (15%) The coil shown in Fig.5 is modeled as a 10mH inductor and a 10Ω resistor in series. Assume the switch has been closed for a long time. Determine the current I and the energy stored in the coil. If the switch is suddenly open, what will happen for the voltage at the node B at that instant (rising, falling or unchanged)?

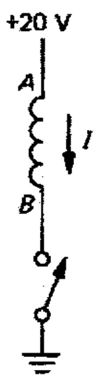


Fig. 5

6. (10%) Two inductive loads are connected in parallel with a 600-V source. The power and current drawn by each load are $P_1 = 23\text{kW}$, $|I_1| = 150\text{A}$ and $P_2 = 60\text{kW}$, $|I_2| = 150\text{A}$, respectively. Calculate the power factor of the combined loads.

(1). Answer the following multiple choice questions: (40%)

1-1. If a coil with an inductive reactance of 10Ω at 120 Hz is connected in series with a capacitor, in order for the circuit to be resonant at 120 Hz, the capacitor should have a capacitive reactance of

(a) 1000Ω
(b) 100Ω
(c) 10Ω
(d) 1Ω

1-2. The power factor of a load is

(a) The sine of the angle between the current and the voltage of the load
(b) Always greater than unity
(c) The ratio of the capacitive reactance and inductive reactance of the load
(d) The ratio of the real (active) power and the volt-amperes of the load

1-3. On a long, high-voltage transmission line under heavy load conditions, var compensation can be provided by installing

(a) Series inductive reactors
(b) Shunt capacitors
(c) Shunt inductive reactors
(d) Series resistors

1-4. Shunt reactors are installed at the terminals of long high-voltage transmission lines in order to

(a) Increase the terminal voltage under heavy load conditions
(b) Compensate for voltage rises caused by capacitive charging currents at light loads
(c) Compensate for the power factor of the connected loads
(d) Increase the phase angle.

1-5. Var are characterized by the fact that they always flow

(a) From points of high voltage to low voltage
(b) From generators to loads
(c) Without effect by the voltages at the line terminals
(d) From points of low voltage to high voltage

1-6. If mechanical input to prime movers of generators of a power system does not match load changes:

(a) System losses will be increased
(b) System frequency and voltage will deviate from normal
(c) System frequency will be low
(d) Nothing will happen

1-7. When a synchronous generator is operating stably and a sudden load change occurs, the rotor will

(a) Advance in its position relative to the stator revolving field
(b) Not be affected
(c) Oscillate in diminishing amounts until it settles to its new correct position
(d) Stop instantly

1-8. Transient stability of a power system is affected by

(a) The characteristics of the generating units
(b) The strength of the transmission network
(c) The speed with which faulted lines can be disconnected
(d) All of the above

1-9. When two conductors are "bundled":

(a) The potential gradient in space is increased.
(b) Corona losses are eliminated.
(c) The potential gradient around them is equivalent to what would exist with one larger-diameter conductor.
(d) The voltages on the two conductors at the same location will be different

1-10. When series capacitors are used in a transmission line:

(a) The phase angle between the sending and receiving ends is reduced under heavy load conditions.
(b) The phase angle between the sending and receiving ends is increased under heavy load conditions.
(c) The phase angle between the sending and receiving ends will be equal.
(d) Line stability is impaired.

(2) For the three-bus transmission network shown in Fig. 1, bus 1 is the slack bus; bus 2 is a load bus; bus 3 is a voltage-controlled bus. The transmission line is assumed lossless.

(a) Find the Y_{bus} of the three bus system (10%)

(b) Derive the three power flow equations used for solving the power flow problem. (10%)

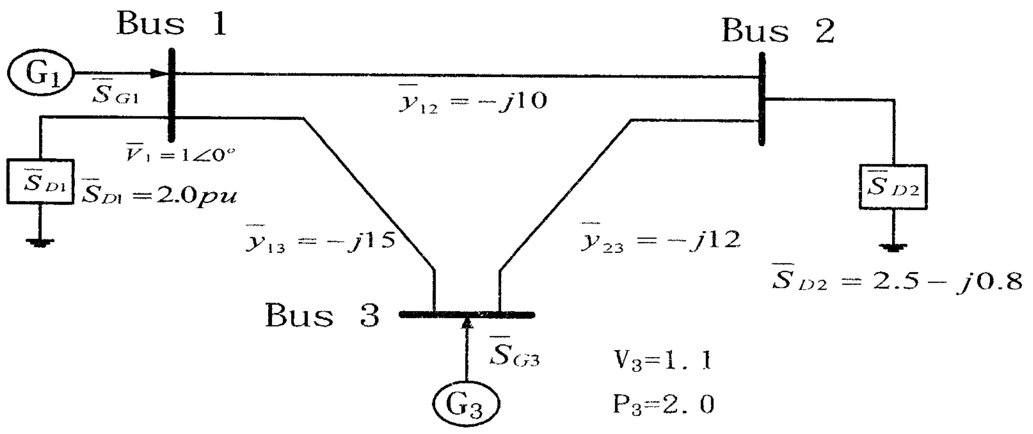


Figure 1

(3) For a section of an electric power system shown in Fig. 2; with Y-Y equipped with primary and secondary neutrals and Y-△ without a primary neutral

(a) Find zero-sequence equivalent circuit of the section of the network (10%)

(b) Find zero-sequence equivalent circuit when transformers at P' are equipped with a neutral connection to ground through Z_n. (10%)

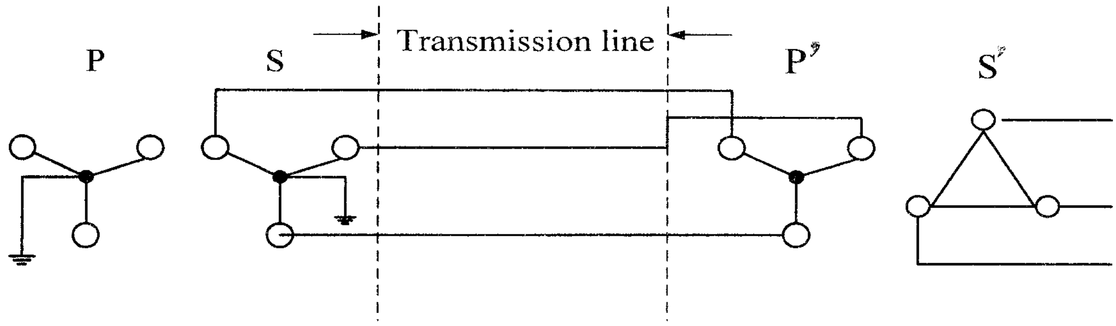


Figure 2

(4) The power needs of a large plant are served by three generating units, which have the following incremental cost functions：

$$IC_1 = \beta_1 + 2\gamma_1 P_{G1} = 8.8 + 0.01 P_{G1} \quad \$/MW-h$$

$$IC_2 = \beta_2 + 2\gamma_2 P_{G2} = 10.2 + 0.015 P_{G2} \quad \$/MW-h$$

$$IC_3 = \beta_3 + 2\gamma_3 P_{G3} = 12.1 + 0.02 P_{G3} \quad \$/MW-h$$

Determine the optimal economic dispatch for a total power demand of P_D = 800 MW. (20%)

1. A spherical dielectric shell of an inner radius r_i and an outer radius r_o is centered at the origin and has a dielectric constant of ε_r. Given a charge distribution

$$\rho_v\ (\text{C/m}^3) = \begin{cases} \rho_0(1 - r^2/r_i^2) & r < r_i \\ 0 & \text{else} \end{cases}, \text{ where } r = \sqrt{x^2+y^2+z^2}, \text{ determine} \qquad (20\%)$$

(a) $\vec{E}$ in $0 \le r < r_o$, (10%)

(b) V and $\vec{P}$ inside the dielectric shell. (10%)

2. (20pts) An air coaxial line with the z-axis as its axis has a hollow inner conductor of radius a and a very thin outer conductor of radius b. Assume a current I flows in the inner conductor and returns in the outer conductor. Denote $\rho = \sqrt{x^2+y^2}$. Calculate (20%)

(a) the magnetic flux density B in $\rho < a$ and $a < \rho < b$, respectively, (10%)

(c) the magnetic energy per unit length stored in the line, (5%)

(d) the inductance per unit length. (5%)

3. Determine the polarization of the following electric fields: (4% each)

(a) $\mathbf{E} = \mathbf{a}_z E_0 \cos(\omega t - \beta y) + \mathbf{a}_x E_0 \sin(\omega t - \beta y)$

(b) $\mathbf{E} = \mathbf{a}_y E_0 \cos(\omega t + \beta x) + \mathbf{a}_z E_0 \sin(\omega t + \beta x)$

(c) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) - \mathbf{a}_z E_0 \sin(\omega t + \beta y)$

(d) $\mathbf{E} = \mathbf{a}_z E_0 \cos(\omega t - \beta x) - \mathbf{a}_y E_0 \sin(\omega t - \beta x + \pi/4)$

(e) $\mathbf{E} = \mathbf{a}_x E_0 \cos(\omega t - \beta y) + \mathbf{a}_z E_0 \cos(\omega t - \beta y)$

4. Consider the partially-filled parallel plate waveguide shown in Fig. P.4. Derive the expressions of electric and magnetic fields inside the waveguide and the cutoff frequency for the TM modes. Can a TEM wave exist in the structure? Ignore fringing fields at the sides with $w >> d$. (20%)

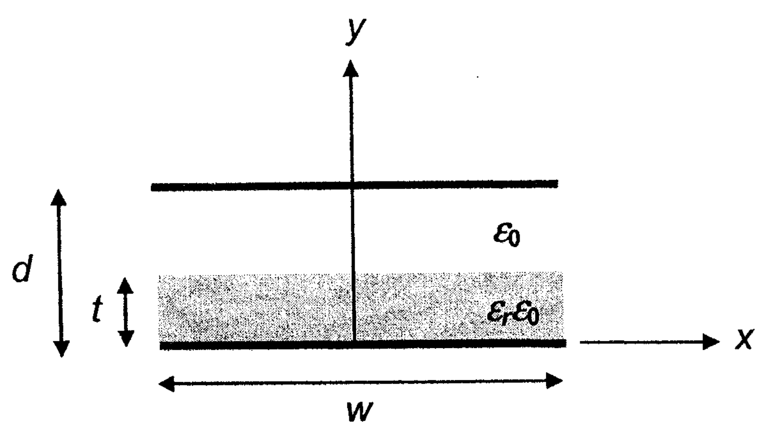


Fig. P4.

5. Consider the quarter-wave impedance matching circuit shown in Fig. P5. Derive the expressions for the amplitude of forward and reverse traveling waves on the quarter-wave line section, V^+ and V^-, in terms of the amplitude of the incident voltage, V_i. (20%)

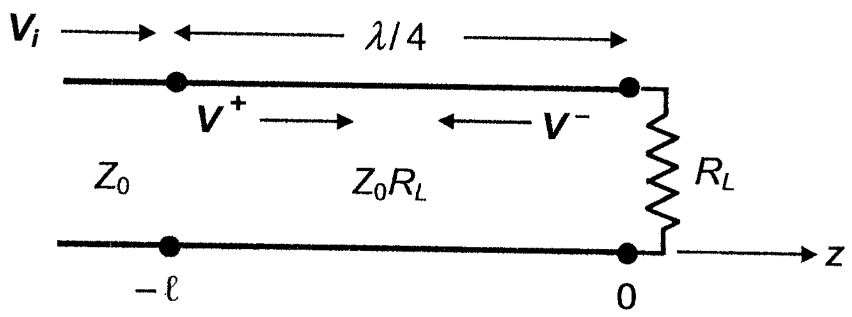


Fig. P5.

1. (15%) For the following matrices

$$A=\begin{bmatrix}4 & -1 & 0 & -1\\ 1 & 4 & 0 & 0\\ 0 & 0 & 4 & 1\\ 2 & 0 & -1 & 4\end{bmatrix}, B=\begin{bmatrix}4 & -1 & 0 & 0\\ -1 & 4 & 0 & 0\\ 0 & 0 & 4 & -1\\ 0 & 0 & -1 & 4\end{bmatrix}, C=\begin{bmatrix}4 & -1 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 4 & -1\\ 0 & 0 & 0 & 4\end{bmatrix}$$

(a) (5%) Determine whether they are orthogonally diagonalizable.

(b) (10%) Find the orthogonal matrices that diagonalize them if they are orthogonally diagonalizable.

2. (5%) Find a matrix A for the linear operator $T:\Re^3 \to \Re^3$ that first rotates a vector counterclockwise about the z-axis though an angle $60°$, then reflects the resulting vector about the yz-plane, and then projects that vector orthogonally onto the xy-plane.

3. (10%) Find QR-decomposition of

$$A=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 3 & 6\\ 0 & 0 & 2 & 4\end{bmatrix}.$$

4. (10%) Find the orthogonal projection of the vector $u=(-1,0,1,3)$ on the subspace of $\Re^4$ spanned by the vectors
$u_1=(3,1,0,2)$, $u_2=(3,6,3,3)$, $u_3=(-2,0,4,-2)$.

5. (10%) Given vectors $u=(1,0,1)$, $v=(1,3,2)$, $w=(0,5,3)$, solve each of the following

(a) (5%) $u\times(v\times w)$.

(b) (5%) $\|w\|^2 u+\|u\|^2 v$.

6. (20%) Consider the following matrix

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & 1 \\ 3 & -1 & 2 \end{bmatrix}.$$

(a)(10%) Find an LU decomposition of the matrix.

(b)(10%) Use LU decomposition to solve the system

$$\begin{aligned} x_1 - x_2 + x_3 &= 4 \\ -x_1 + 2x_2 + x_3 &= -1 . \\ 3x_1 - x_2 + 2x_3 &= 8 \end{aligned}$$

7. (20%) Suppose that $T : \Re^2 \to \Re^3$ is defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 - 4x_2 \\ -x_1 + 2x_2 \\ 5x_1 \end{bmatrix}.$$

(a) (7%) Determine a spanning set for the range of T.

(b) (7%) Determine a spanning set for the null space of T.

(c) (2%) Is T onto?

(d) (2%) Is T one-to-one?

(e) (2%) Is T invertible?

8. (10%) Let T be a linear operator on $\Re^3$ such that

$$T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}.$$

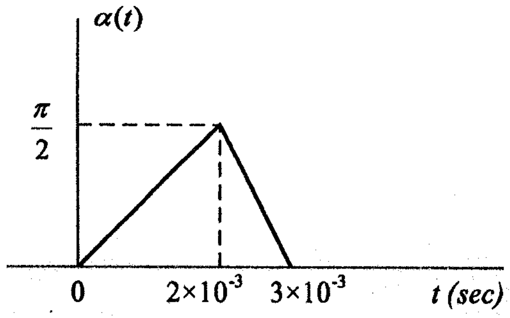
Find the standard matrix of T.

通訊理論 (Communications Theory)

1. (15%) Let $x(t) = t^{-0.25}$, $t \geq t_0 > 0$, and zero otherwise. Compute the energy and power in $x(t)$, and determine whether $x(t)$ is an energy-type signal or a power-type signal.

2. (15%) Find the Hilbert Transform $\widehat{x}(t)$ of a signal $x(t) = \cos(\omega_0 t) + \sin(\omega_0 t)$. Based on your result, determine whether $\widehat{x}(t)$ and $x(t)$ are orthogonal.

3. (15%) Consider the angle modulated wave $cos(\omega_c t + \alpha(t))$, where $\alpha(t)$ is shown below.



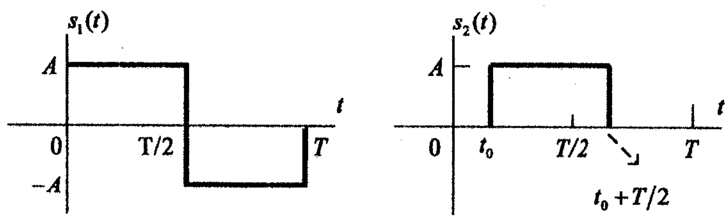
   (a) (5%) What is the maximum frequency deviation?

   (b) (10%) Now consider the composite wave

$$e(t) = cos\omega_c t + cos(\omega_c t + \alpha(t)).$$

   Let $e(t) = \text{Re}\{a(t)e^{j\phi(t)}e^{j\omega_c t}\}$. Draw the locus of $a(t)e^{j\phi(t)}$ for the time interval $0 \leq t \leq 3 \times 10^{-3}$ seconds. What is the maximum of the phase deviation $\phi(t)$?

4. (20%) A pair of pulses are shown below.



   (a) (6%) Find the optimum (matched) filter impulse response $h_0(t)$ for $s_1(t)$ and $s_2(t)$.

(b) (8%) What is the best choice for t_0 such that the error probability at the receiver is minimized? Why?

(c) (6%) Sketch a correlator receiver structure for these signals.

5. (20%)

(a) (6%) What are the differences between source coding and channel coding?

(b) (6%) What is Sampling Theorem? Describe the theorem, applications and drawbacks.

(c) (8%) List two types of degradation from which the error performance of digital signaling suffers. What are the typical solutions to dealing with the error sources?

6. (15%) Twenty-five audio input signals, each bandlimited to $3.5kHz$ and sampled at a $10kHz$ rate, are time-multiplexed in a PAM system.

(a) (7%) Determine the minimum clock frequency of the system.

(b) (8%) Find the maximum pulse width for each channel.

~End~

1. Let X_1 and X_2 be two continuous random variables with joint probability density function

$$f_{X_1X_2}(x_1,x_2)=\begin{cases}4x_1x_2, & if \quad 0<x_1<1, \quad 0<x_2<1 \\ 0 \ , & otherwise\end{cases}$$

(a) Find $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$. (10%)

(b) Find the joint probability density function of Y_1 and Y_2 , $f_{Y_1Y_2}(y_1, y_2)$, where $Y_1 = X_1^2$ and $Y_2 = X_1X_2$, (8%)

(b) Find $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$ (7%)

2. Consider a communication channel corrupted by noise. Let random variable X be the value of the transmitted signal and Y be the value of the received signal. Assume that the conditional density of Y is given $\{X = x\}$ is Gaussian, i.e.

$$f_{Y|X}(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp[-\frac{(y-x)^2}{2\sigma^2}]$$

and X is uniformly distributed on [-1, 1].

(a) What is the probability density function of Y, $f_Y(y)$ (7%)

(b) What is the conditional density of X is given Y (*i.e.* $f_{X|Y}(x|y)$)? (8%)

3. A zero-mean normal (*Gaussian*) random vector $\mathbf{X} = (X_1, X_2)^T$ has *covariance matrix* $\mathbf{K} = E[\mathbf{X}\mathbf{X}^T]$, which is given by

$$\mathbf{K} = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \qquad (10\%).$$

Find a transformation $\mathbf{Y}=\mathbf{D}\mathbf{X}$ such that $\mathbf{Y}= (Y_1, Y_2)^T$ is a normal (or Gaussian) random vector with uncorrelated (and therefore *independent*) components of unity variance

4. Consider two *independent identical distribution* (*i.i.d*) *random variables*, X and Y.

(a) Find the *probability density function* of *random variable*, Z=X+Y. (7%)

Now, if the *probability density functions* of X and Y are with

$$f_X(x) = f_Y(x) = \frac{1}{a}rect\,(\frac{x}{a}) = \begin{cases}\frac{1}{a} & ,-\frac{a}{2} \le x \le \frac{a}{2} \\ 0 & , \quad otherwise\end{cases}$$

(b) Find the *Characteristic function* of X and Z, where $\Phi_X(\omega) = E[e^{j\omega X}]$. (8%)

(c) Compute the *probability density function* of Z. (5%).

5. Let X_1 and X_2 be two independent *Poisson* random variables with identical distribution.

(a) Find $P[X_1 = x_1 \mid X_1 + X_2 = y]$ (8%)

(b) Find $E[X_1 \mid X_1 + X_2 = y]$ (7%)

6. Assume that *random variable X* has a *gamma distribution* with *probability density function*, which is defined by

$$f_X(x) = \begin{cases} \dfrac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$. The *gamma function* $\Gamma(\alpha)$ is defined by

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$$

for $\alpha > 0$ and has the following properties, e.g., $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$, $\Gamma(n+1) = n!$, $\Gamma(1/2) = (\pi)^{1/2}$ and $\Gamma(1)=1$.

(a) For $\alpha = 1/2$ and $\beta = 2$, please evaluate the *mean* $\mu = E[X]$, and *variance* $\sigma_X^2 = E[(X - E[X])^2]$ (8%).

(b) For $\alpha = \upsilon/2$ and $\beta = 2$, we have the so-called *chi-square distribution*, again, find the *mean* μ and σ_X^2 for *random variable X* (7%).

(*Note:* υ is the degree of freedom and $\upsilon > 0$)