

## 國立中山大學100學年度碩士班招生考試試題

科目：計算機結構【資工系碩士班】

1. (12% total) The table below shows the percentage of MIPS instructions executed by category for average of five SPEC2000 integer programs and five SPEC2000 floating-point programs.

Instruction	MIPS examples	Average CPI	Frequency	
			Integer	Floating-point
Arithmetic	add, sub, addi	1.0 clock cycles	24%	48%
Data transfer	lw, sw, lb, sb, lui	1.4 clock cycles	36%	39%
Logical	and, or, nor, andi, ori	1.0 clock cycles	18%	4%
Conditional branch	beq, bne, slt, slti	1.7 clock cycles	18%	6%
Jump	j, jr, jal	1.2 clock cycles	4%	2%

- 1.1 (3%) Using the information for the program SPEC2000int, find the percentage of all memory accesses (both data and instruction) that are for data.
- 1.2 (4%) Using the information for the program SPEC2000fp, find the percentage of all memory accesses (both data and instruction) that are for reads. Assume that two-thirds of data transfers are loads.
- 1.3 (5%) Compute the effective CPI for MIPS. Average the instruction frequencies for SPEC2000int and SPEC2000fp to obtain the instruction mix.

2. (10% total) The 32-bit pattern  $0xA4180000_{\text{sixteen}}$  is expressed in hexadecimal notation. What decimal number does this bit pattern represent?
- 2.1 (4%) Assume that it is a two's-complement integer.
- 2.2 (6%) Assume that it is a single precision IEEE 754 floating-point number.

3. (12% total) Consider the computer with three instruction classes and the following CPI measurements:

	CPI for this instruction class		
	A	B	C
CPI	1	2	3

Now suppose we measure the code for the same program from two different compilers and obtain the following data:

Code from	Instruction counts (in billions) for each instruction class		
	A	B	C
Compiler 1	5	1	1
Compiler 2	11	1	1

Assume that the computer's clock rate is 4 GHz.

- 3.1 (6%) Compute the MIPS (million instructions per second) rate for each version of the program. Which code sequence will execute faster according to MIPS?
- 3.2 (6%) Find the execution time for each version of the program. Which code sequence will execute faster according to execution time?

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4. (20% total) Compare performance for single-cycle, multicycle, and five-stage (IF, ID, EX, MEM, WB) pipelined control using the following instruction mix: 25% loads, 10% stores, 11% branches, 2% jumps, and 52% ALU instructions. Assume that the operation times for the major functional units are the following:

- Memory units: 200 picoseconds (ps)
- ALU and adders: 100 ps
- Register file (read or write): 50 ps

In the multicycle implementation, the number of clock cycles required to execute each instruction class is 5, 4, 4, 3, and 3 for loads, stores, ALU instructions, branches, and jumps, respectively.

For pipelined execution, assume that half of the load instructions are immediately followed by an instruction that uses the result, that the branch delay on misprediction is 1 clock cycle, and that one-quarter of the branches are mispredicted. Assume that jumps always pay 1 full clock cycle of delay, so their average time is 2 clock cycles. Ignore any other hazards.

- 4.1 (3%) What is the clock cycle time for single-cycle, multicycle, and pipelined datapath?  
 4.2 (10%) What is the CPI for single-cycle, multicycle, and pipelined design?  
 4.3 (4%) What is the average instruction time for single-cycle, multicycle, and pipelined design?  
 4.4 (3%) If we can split one stage of the pipelined datapath into two new stages, each with half the operation time of the original stage to shorten the clock cycle time, which stages would you split and what is the new clock cycle time of the pipelined datapath?

5. (12% total) Consider executing the following code on a five-stage (IF, ID, EX, MEM, WB) pipelined datapath:

```
add    $3, $4, $2
sub    $5, $3, $1
lw     $6, 200($3)
add    $7, $3, $6
```

- 5.1 (3%) Identify all of the data dependencies.  
 5.2 (2%) Which dependencies are data hazards that will be resolved via forwarding?  
 5.3 (2%) Which dependencies are data hazards that will cause a stall?  
 5.4 (5%) How many cycles will it take to execute this code?

6. (12% total) Assume an instruction cache miss rate for a program is 2% and a data cache miss rate is 4%.

- 6.1 (6%) If a processor has a CPI of 2 without any memory stalls and the miss penalty is 100 cycles for all misses, determine how much faster a processor would run with a perfect cache that never missed. Use the instruction frequencies for SPEC2000int shown in Problem 1.  
 6.2 (6%) Suppose we increase the performance of the processor by doubling its clock rate. Since the main memory speed is unlikely to change, assume that the absolute time to handle a cache miss does not change. How much faster will the processor be with the faster clock, assuming the same miss rate as Problem 6.1?

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7. (10%) Increasing associativity requires more comparators, and more tag bits per cache block. Assuming a cache of 4K blocks, a four-word block size, and a 32-bit address, find the total number of sets and the total number of tag bits for caches that are direct mapped, two-way and four-way set associative, and fully associative.
8. (12% total) Please briefly answer the following questions.
- 8.1 (4%) Suppose you want to achieve a speedup of 80 with 100 processors. What fraction of the original computation can be sequential?
- 8.2 (4%) What's the main idea of loop unrolling? List at least two aspects that the loop unrolling can help to improve performance.
- 8.3 (4%) What are the dynamic branch prediction and the branch prediction buffer?

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## [Process Scheduling 10%]

1. Consider a preemptive priority scheduling algorithm based on dynamically changing priorities. Larger priority numbers imply higher priority. When a process is waiting for the CPU (in the ready queue, but not running), its priority changes at a rate  $A$ ; when it is running, its priority changes at a rate  $B$ . All processes are given a priority of 0 when they enter the ready queue. The parameters  $A$  and  $B$  can be set to give many different scheduling algorithms.

- (a). What is the algorithm that results from  $A < B < 0$ ? (5%)  
 (b). What is the algorithm that results from  $B > A > 0$ ? (5%)

## [Virtual-Memory Management 15%]

2. Assume we have a demand-paged memory. The page table is held in registers. It takes 8 milliseconds to service a page fault if an empty page is available or the replaced page is not modified, and 20 milliseconds if the replaced page is modified. Memory access time is 100 nanoseconds. Assume that the page to be replaced is modified 70 percent of the time. What is the maximum acceptable page-fault rate for an effective access time of no more than 200 nanoseconds?

## [Deadlocks 15%]

3. Consider the dining-philosophers problem where the chopsticks are placed at the center of the table and any two of them could be used by a philosopher. Assume that requests for chopsticks are made one at a time. Describe a simple rule for determining whether a particular request could be satisfied without causing deadlock given the current allocation of chopsticks to philosophers.

## [Memory-Management Strategies 15%]

4. Consider the following segment table:

Segment	Base	Length
0	219	700
1	2315	15
2	100	90
3	1327	580
4	1917	100

What are the physical addresses for the following logical addresses?

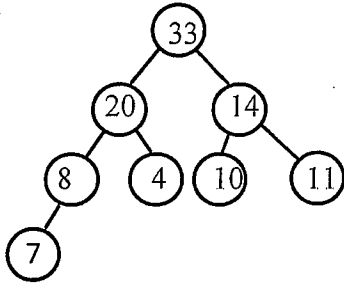
- (a). Segment 4, byte 50. (3%)  
 (b). Segment 1, byte 20. (3%)  
 (c). Segment 3, byte 500. (3%)  
 (d). Segment 2, byte 400. (3%)  
 (e). Segment 0, byte 112. (3%)

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[Search &amp; Sort 20%]

5. (a). Is this tree a Max Heap? Explain why. (10%)  
 (b). Assume the tree is Max Heap, Draw the Max Heap after deleting Max element twice. (10%)

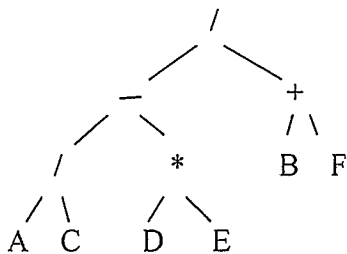


[Hash Method 10%]

6. Use  $(x \text{ MOD } 8)$  to be the hashing function and show the final hash table with linear probing ( where every bucket has only one slot ) after inserting keys 10, 32, 15, 58, 18, 21, 9 into an initially empty hash table.

[Tree 15%]

7. Use the expression tree




- (a). find postfix expressions. (5%)  
 (b). find infix expressions. (5%)  
 (c). find prefix expressions. (5%)

## 國立中山大學100學年度碩士班招生考試試題

科目：離散數學【資工系碩士班甲組】

There are 8 problems in this test. Write down detailed steps for the solution to each problem. Otherwise, no credits for that problem will be given.

1. (10%) Let  $m > 1$  and  $n > 1$  be two positive integers. Let  $r(m, n)$  denotes the maximum number of rectangles defined by  $m$  horizontal lines and  $n$  vertical lines in a plane. Derive a formula for  $r(m, n)$ . Note that rectangles may overlap. For example, let  $m = 2$  and  $n = 3$  (  ),  $r(2, 3) = 3$ , not 2.
2. (10%) Let  $x_1, x_2, \dots, x_n$  be a sequence of  $n$  integers. A consecutive subsequence of  $x_1, x_2, \dots, x_n$  is a subsequence  $x_i, x_{i+1}, \dots, x_j$  for some  $1 \leq i \leq j \leq n$ . Show that for any  $k$ ,  $1 \leq k \leq n$ , there is a consecutive subsequence whose sum is divisible by  $k$ .
3. (10%) Let the sequence of numbers  $g_0, g_1, \dots, g_n, \dots$  be defined by  $g_0 = 1$ ,  $g_1 = 1$  and, for every  $n > 1$ ,  $g_n = g_{n-1} + 2g_{n-2} + (-1)^n$ . Express  $g_n$  in terms of  $n$ .
4. (10%) A *planar* graph is a graph which can be embedded in a plane without crossing edges. Let  $G = (V, E)$  be a simple graph with  $n$  vertices and  $m$  edges.
  - (a) (5%) Show that if  $G$  is planar with  $n > 2$ , then  $m \leq 3n - 6$ .
  - (b) (5%) Define  $G^c$ , the complement of  $G$ , to be the graph with the vertex set  $V$ . For every pair of vertices  $x$  and  $y$ , the edge  $xy$  is in  $G^c$  if and only if  $xy$  is not in  $G$ . Show that if  $G$  is planar with  $n > 10$ , then  $G^c$  is not planar.
5. (10%) Simplify the Boolean function  $(f + g + h)(f + g + \bar{h})(f + \bar{g} + h)$ , by using
  - (a) (5%) the laws of Boolean algebra, and
  - (b) (5%) the method of Karnaugh maps.
6. (10%) A *tree* is a connected graph without cycles.
  - (a) (5%) Show that if  $G$  is a tree of more than 1 vertex, then  $G$  has at least 2 vertices of degree 1.
  - (b) (5%) Can there be a stronger theorem:  
If  $G$  is a tree of more than  $\alpha$  vertices, then  $G$  has at least  $\beta$  vertices of degree 1.  
for some  $\alpha$  and  $\beta > 2$ . Justify your answer.

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7. (20%) Let  $S = \{1, 2, \dots, n\}$ . Define a relation  $\sim$  on  $S$ , such that  $x \sim y$  if and only if  $x = 2^k y$  for some integer  $k$ .
- (a) (5%) Show that the relation  $\sim$  is an equivalence relation.
- (b) (5%) Show the equivalence classes for  $n = 20$  and  $n = 25$ , respectively.
- (c) (10%) Let  $\lfloor x \rfloor$  be the largest integer less than or equal to  $x$ . Show that if  $\lfloor \frac{n+1}{2} \rfloor + 1$  numbers are chosen from the set  $S$ , then there must be two numbers  $a$  and  $b$  such that  $a$  is divisible by  $b$ .
8. (20%) Let  $m$  and  $n$  be two positive integers,  $m \leq n$ . Define  $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ .
- (a) (10%) Show that if  $n$  is prime, then  $n$  divides  $\binom{n}{i}$  for every  $i$ ,  $1 \leq i < n$ .
- (b) (10%) Show that if  $n$  is composite, then  $n$  does not divide  $\binom{n}{i}$  for some  $i$ ,  $1 \leq i < n$ .

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科目：工程數學【資工系碩士班乙組】

1. (10%) Solve the following exact differential equation:

$$(y^2 + 1)dx + (2xy + 4)dy = 0$$

2. (15%) Laplace Transform

- 2.1 (10%) Solve the following equation by Laplace transform:

$$y(t) = t + \int_0^t y(t - \tau) \sin \tau d\tau$$

- 2.2 (5%) Find the inverse Laplace transform of

$$F(s) = \frac{s}{(s - 2)(s^2 + 4s + 5)}$$

3. (25%)

- 3.1 (10%) Given  $f(x)=x$ ,  $g(x)=x^2$ , find the *Inner Product* and *norms* of  $f(x)$  and  $g(x)$  on the interval  $0 \leq x \leq 1$ .

- 3.2 (5%) Let  $\mathbf{B} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -3 & 1 \\ 1 & 3 & -3 \end{bmatrix}$ , find  $\mathbf{B}^{-1}$ .

- 3.3 (5%) Find the rank of matrix  $\mathbf{B}$ ,  $\text{Rank}(\mathbf{B})$ , where  $\mathbf{B} = \begin{bmatrix} 3 & 1 & 4 & 0 \\ 1 & 0 & 1 & -2 \\ 2 & 1 & 3 & 2 \end{bmatrix}$ .

- 3.4 (5%) An  $n \times n$  matrix  $\mathbf{B}$  is orthogonal if  $\mathbf{B}^{-1} = \mathbf{B}^T$ . Determine whether  $\mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$  is

orthogonal and find  $\mathbf{B}^{-1}$ .

4. (20%) Let  $\mathbf{B} = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$  and  $f(x) = x^5 - 3x^4 + 4x^3 - 2x^2 + 6x - 2$ .

- 4.1. (10%) Find  $f(\mathbf{B})$ .

- 4.2. (5%) Find the eigenvalues of  $f(\mathbf{B})$ .

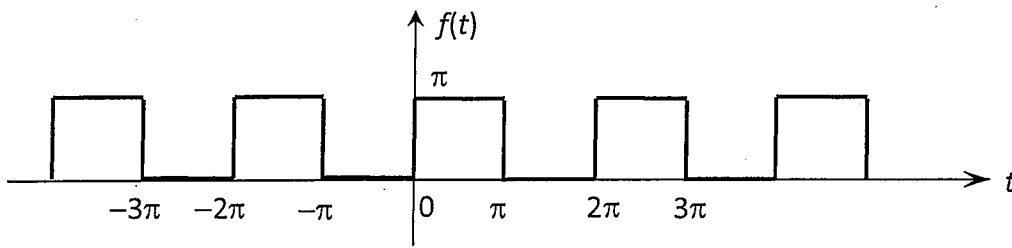
- 4.3. (5%) What is the determinant of  $f(\mathbf{B})$  (i.e.,  $|f(\mathbf{B})|$ )?

5. (20%) Fourier Analysis

- 5.1 (10%) Find the Fourier coefficients of the periodic function

$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ \pi & 0 < t < \pi \end{cases}$$





5.2 (5%) The Fourier transform  $X(\omega)$  of  $x(t)$  is defined as  $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$ .

Find the Fourier transform of the following function

$$x(t) = \begin{cases} 0 & t < -1 \\ 1 & -1 < t < 1 \\ 0 & t > 1 \end{cases}$$

5.3 (5%) The inverse Fourier transform  $x(t)$  of  $X(\omega)$  is defined as

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega. \text{ Use the result of Problem 5.2, prove that}$$

$$\int_0^{\infty} \frac{\sin \omega}{\omega} d\omega = \frac{\pi}{2}.$$

6 (10%) z-Transform

The z-transform of a discrete data sequence  $x[n] = \{x_0, x_1, \dots\}$  is defined as:

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}$$

6.1 (3%) Derive the z-transform of sequence  $x[n]$  where  $x_n = b^n$  and  $b$  is a constant.

6.2 (7%) Let z-transform of a sequence  $y[n]$  be denoted as  $Z\{y[n]\}$ . Show that  $Z\{y[n-1]\} = z^{-1} \cdot Z\{y[n]\}$ , and solve the following difference equation by using z-transform:

$$y[n] - \frac{1}{2}y[n-1] = \delta[n]$$

where  $\delta[n]$  is the delta function.