# 國立中山大學99學年度碩士班招生考試試題

科目：計算機結構【資工系碩士班】

1. (15% total) You are going to enhance a machine, and there are two possible improvements: either make multiply instructions run four times faster than before, or make memory access instructions run two times faster than before. You repeatedly run a program that takes 100 seconds to execute. Of this time, 20% is used for multiplication, 50% for memory access instructions, and 30% for other tasks.

1.1. (5%) What will the speedup be if you improve only multiplication?

1.2. (5%) What will the speedup be if you improve only memory access?

1.3. (5%) What will the speedup be if both improvements are made?

2. (10%) In this problem, we'll examine quantitatively the pros and cons of adding an addressing mode to MIPS that allows arithmetic instructions to directly access memory, as is found on the 80x86. The primary benefit is that fewer instructions will be executed because we won't have to first load a register. The primary disadvantage is that the cycle time will have to increase to account for the additional time to read memory. Consider adding a new instruction:

```
addm $t2, 100($t3)    # $t2 = $t2 + Memory [ $t3 + 100 ]
```

Assume that the new instruction will cause the cycle time to increase by 10%. Use the instruction frequencies for the GCC benchmark, and assume that two-thirds of the data transfers are loads and the rest are stores. Assume that the new instruction affects only the clock time, not the CPI. What percentage of loads must be eliminated for the machine with the new instruction to have at least the same performance?

| Instruction class | MIPS examples | High level language correspondence | Frequency of GCC Benchmark |
|---|---|---|---|
| Arithmetic | add, sub, addi | Operations in assignment statements | 48% |
| Data transfer | lw, sw, lb lui | Reference to data structure, such as arrays | 33% |
| Conditional branch | beq, bne, slt, slti | *if* statements and loop | 17% |
| Jump | j, jr, jal | Procedure calls, returns and case/switch statements | 2% |

3. (10%) Determine whether it is possible for overflow to occur for the following operations under the different binary encoding methods. If overflow is possible, provide an example. Assume that both A and B are four-bit binary numbers, and that it is known that A=0. Please draw the following table in your answer sheet.

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| Binary encoding | 2's complement | | | | |
|---|---|---|---|---|---|
| Operation | A+B | A-B | A+B+1 | A-B+1 | A+B-1 |
| Overflow? (yes/no) | | | | | |
| Example for overflow (B=...) | | | | | |

4. (15% total) Consider a three-bit adder with its two operands $A_i$ and $B_i$ where $i$=0…2 (0 is the least significant bit) and the carry-in bit $C_0$ and carry-out bit $C_3$.

4.1. (5%) Derive the Boolean equation for the carry-out bit $C_3$ using only $A_i$, $B_i$ (where $i$=0…2) and $C_0$.

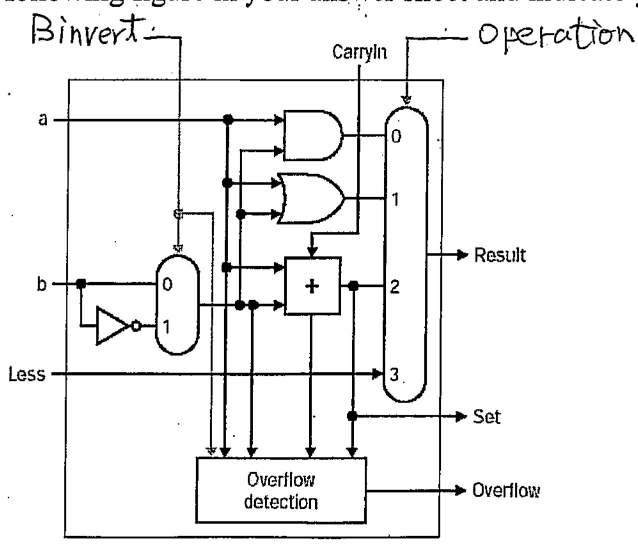
4.2. (5%) In a carry-lookahead adder, there are two additional signals defined, $G_i=A_i \cdot B_i$ (generate), $P_i=A_i+B_i$ (propagate). Derive the Boolean equation for the carry-out bit $C_3$ using only $G_i$, $P_i$ (where $i$=0…2) and $C_0$.

4.3. (5%) What are the advantages of the Boolean equation in Problem 4.2 over the Boolean equation in Problem 4.1?

5. (10%) The ALU supported set on less than (slt) using just the sign bit of the adder. Let's try a set on less than operation using the value $-7_{ten}$ and $6_{ten}$. To make it simpler to follow the example, let's limit the binary representations to 4 bits: $1001_{two}$ and $0110_{two}$

$$1001_{two} - 0110_{two} = 1001_{two} + 1010_{two} = 0011_{two}$$

This result would suggest that –7 > 6, which is clearly wrong. Hence we must factor in overflow in the decision. Modify the 1-bit ALU to handle *slt* correctly. Draw the following figure in your answer sheet and indicate your modification.




6. (10%) The MIPS processor has a 32-bit memory address (in byte). The following is a direct-map cache for the MIP processor. This cache has 4K entries (or called blocks, lines, *etc.*) with each entry containing 4 memory words (one word is equivalent to four bytes). The following is the figure of this cache.

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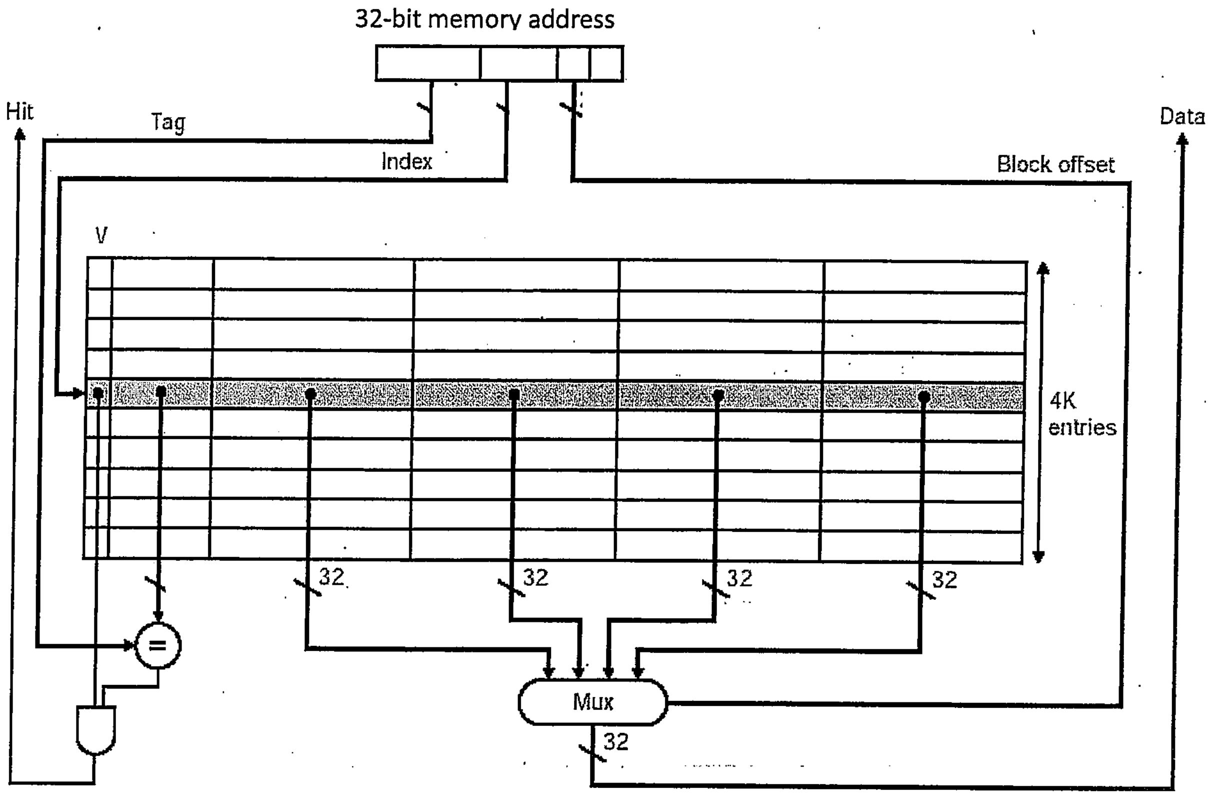
6.1. (5%) What's the bit width of the tag?
6.2. (5%) What's the bit width of the index?




7. (10%) There are three small caches, each consisting of four one-word blacks. One cache is fully associative, a second is two-way set associative, and the third is direct mapped. Find the numbers of misses for each organization given the following sequence of block address: <u>0, 8, 0, 6, 8.</u> Assume that the least recently used replacement policy is used for the fully associative cache and the set-associative cache.

8. (5%) What is the average time to read or write a 512-byte sector for a typical disk rotating at 5400 RPM? The advertised average seek time is 12 ms, the transfer rate is 5 MB/sec, and the controller overhead is 2 ms. Assume that the disk is idle so that is no waiting time.

9. (Total 15%) Suppose we want to sum 100,000 numbers on a single-bus multiprocessor computer. Let's assume we have 10 processors. The first step would be to split the set of numbers into subsets of the same size. All processors start the program by running the following loop that sums their subset of numbers:

```
sum[Pn] = 0;
for (I = 10000 * Pn; I < 10000 * (Pn+1); I = I + 1)
    sum[Pn] = sum[Pn] + A[I]; // sum the assigned areas
```

The next step is to add these many partial sums, so we divide to conquer. Half of processors add pairs of partial sums, then a quarter add pairs of the new partial sums, and so on until we have the single, final sum. We want each processor to have its own version of loop counter variable *I*, so we must indicate that it is a "private" variable.

In this problem, the two processors must synchronize before the "consumer" processor tries to read the result from the memory location written by the "producer" processor; otherwise, the consumer may read the old value of the data. Here is the code (*half* is private also):

```
half = 10; //10 processors in 1-bus multiprocessor.
repeat
        synch(); // wait for partial sum completion.
        if (half%2 != 0 && Pn == 0)
            sum[0] = sum[0] + sum[half - 1];
        half = half / 2; // dividing line on who sums.
        if (Pn < half) sum[Pn] = sum[Pn] + sum[Pn+half];
until (half == 1); // exit with final sum in sum[0].
```

Question: according to the algorithm, find out what operations are executed by the designated processor during the designated repeat-loop iteration. (Ex: NOP or sum[0] = sum[0] + sum [4])

9.1. (5%) Processor Pn = 1, during the first iteration.

9.2. (5%) Processor Pn = 5, during the second iteration.

9.3. (5%) Processor Pn = 0, during the third iteration.

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## 科目：作業系統與資料結構【資工系碩士班甲組】

1. [Multiple choices] There may be zero or more correct answers in each of the following problems. If there is no correct answer, you should give the answer "None". (20%)
   (a) Which sorting algorithm(s) is a stable sorting method? (A) Bubble sort (B) Insertion sort (C) Merge sort (D) Quicksort sort.
   (b) Which sorting algorithm(s) has time complexity O(n) in the best case? (A) Bubble sort (B) Insertion sort (C) Merge sort (D) Quicksort sort.
   (c) Which statement(s) is correct for an AVL tree? (A) The height difference of two subtrees of each node is at most one. (B) The sizes (number of nodes) of the two subtrees of each node differ at most one. (C) The level difference of any pair of leaves is at most one. (D) After a rotation is performed, the sequence of preorder traversal is preserved.
   (d) Which statement(s) is correct for hashing? (A) The memory allocated should not be less than the requirement for inserting data elements. (B) Hashing is a fast sorting method. (C) Hash collisions may occur when data elements are inserted. (D) An element deletion can be accomplished by first finding its position with the hashing function and then removing the element directly.
   (e) Which statement(s) is correct? (A) A stack can be implemented by arrays. (B) A stack can be implemented by linked lists. (C) A linked list can be implemented by arrays. (D) A linked list can be implemented by stacks.

2. [Expression] Suppose we have a prefix expression +*-ABC*/DE*+FGH.
   (a) Draw the binary expression. (4%)
   (b) Give the infix expression and postfix expression. (6%)

3. [Recursion] There is a complete binary tree with $L$ levels, where $L$=1 when the tree consists of only one node. Each node of the tree (leaf and internal node) stores one integer element, where the elements stored in the tree are unordered. Someone writes a recursive C function to find the maximum element in the tree as follows. *max*( ) is called by passing the root of the tree as the parameter. (20%)

```
sturct nodetype {
    int d;
    struct nodetype *left;
    struct nodetype *right;
}
int max(struct nodetype *p)
{
   if(p==NULL)
      return(-MAXINT); //MAXINT is infinite
   else {
     printf(%d \n,p->d);
     if (max(p->left) >= max(p->right)
        return (p->d >= max(p->left) ? p->d : max(p->left))
     else
        return (p->d >= max(p->right) ? p->d : max(p->right))
   }
}   /* end of max() */
```

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科目：作業系統與資料結構【資工系碩士班甲組】

(a) How many lines are printed in the worst case? Please derive your answer.
(b) How many lines are printed in the best case? Please derive your answer.

4. [CPU Scheduling] Consider a system running ten I/O-bound tasks and one CPU-bound task. Assume that the I/O-bound tasks issue an I/O operation once for every millisecond of CPU computing and that each I/O operation takes 10 milliseconds to complete. Also assume that the context switching overhead is 0.1 milliseconds and that all processes are long-running tasks. What is the CPU utilization for a Round-robin scheduler when :
   (a) The time quantum is 1 millisecond (5%)
   (b) The time quantum is 10 milliseconds (5%)

5. [Deadlocks] Consider a system consisting of $m$ resources of the same type, being shared by $n$ processes. Resources can be requested and released by processes only one at a time. Show that the system is deadlock free if the following two conditions hold. (10%)
   (a) The maximum need of each process is between 1 and $m$ resources.
   (b) The sum of all maximum needs is less than $m + n$.

6. [Memory Management] Consider a paging system with the page table stored in memory.
   (a) If a memory reference takes 200 nanoseconds, how long does a paged memory reference take? (5%)
   (b) If we add associative registers, and 75 percent of all page-table references are found in the associative registers, what is the effective memory reference time? (Assume that finding a page-table entry in the associative registers takes zero time, if the entry is there.) (5%)

7. [Virtual Memory] A certain computer provides its users with a virtual-memory space of $2^{32}$ bytes. The computer has $2^{18}$ bytes of physical memory. The virtual memory is implemented by paging, and the page size is 4096 bytes. A user process generates the virtual address 11123456. Explain how the system establishes the corresponding physical location. Distinguish between software and hardware operations. (10%)

8. [File-System Implementation] Consider a file system on a disk that has both logical and physical block sizes of 512 bytes. Assume that the information about each file is already in memory. For each of the three allocation strategies (contiguous, linked, and indexed), answer these questions:
   (a) How is the logical-to-physical address mapping accomplished in this system? (For the indexed allocation, assume that a file is always less than 512 blocks long.) (5%)
   (b) If we are currently at logical block 10 (the last block accessed was block 10) and want to access logical block 4, how many physical blocks must be read from the disk? (5%)

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科目：離散數學【資工系碩士班甲組】

1. Determine the number of integer solutions of the following equations:
   (a) [5%] $x_1 + x_2 + x_3 + x_4 = 17$ where $x_i \geq 0$ for $i = 1, 2, 3$, and 4.
   (b) [5%] $x_1 + x_2 + x_3 + x_4 + x_5 = 16$ where $x_i > 0$ for $i = 1, 2, 3, 4$, and 5.
   (c) [5%] $x_1 + x_2 + 9x_3 + x_4 + x_5 = 19$ where $x_i \geq 0$ for $i = 1, 2, 3, 4$, and 5.

2. Let $\Sigma$ be an alphabet.
   (a) [5%] If $\Sigma = \{1, 2, 3, 4\}$, what is $|\Sigma^3|$?
   (b) [5%] If $\Sigma = \{a, b, c, d, e\}$, how many strings in $\Sigma^*$ have length at most 5?
   (c) [5%] If $\Sigma = \{ab, c, de\}$, what is the length of the string *abcde*?

3. [10%] Find the coefficient of $x^{72}$ in $(x^6 + x^7 + x^8 + \ldots)^{10}$.

4. Let $A$ be a set with $|A| = n$, and let $R$ be a binary relation on $A$ that is reflexive and antisymmetric.
   (a) [5%] What is the maximum value for $|R|$?
   (b) [5%] How many antisymmetric relations can have this size?

5. Let $A$ be a set with $|A| = n$.
   (a) [5%] If $n = 5$, how many binary relations on $A$ are symmetric but not reflexive? (Your final answer should be an integer, not a formula.)
   (b) [5%] If $n = 4$, how many binary relations on $A$ are neither reflexive nor symmetric? (Your final answer should be an integer, not a formula.)

6. [10%] Use Chinese Remainder Theorem to find a simultaneous solution for the system of three congruences: $x \equiv 5 \pmod{15}$, $x \equiv 4 \pmod{14}$, $x \equiv 0 \pmod{13}$ where $2500 < x < 4500$.

7. (a) [5%] How many units are there in $Z_{200}$?
   (b) [5%] Solve $x^2 \equiv 1 \pmod{15}$ in $Z_{15}^*$.

8. [10%] Let $G = (V, E)$ be a connected planar graph or multigraph with $|V| =$ 2000 and $|E| = 2500$. What is the number of regions in the plane determined by a planar embedding (or, depiction) of $G$.

9. Solve the following recurrence relations:
   (a) [5%] $a_n - 5a_{n-1} - 6a_{n-2} = 0$, $n \geq 2$, $a_0 = 0$, $a_1 = 1$.
   (b) [5%] $a_{n+1} - 2a_n = 2^n$, $n \geq 0$, $a_0 = 2$.

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科目：工程數學【資工系碩士班乙組】

工程數學

1. (15%) Let $A=\begin{pmatrix} 5 & -4 & 4 \\ 12 & -11 & 12 \\ 4 & -4 & 5 \end{pmatrix}$.

1.1 (5%) Find the eigenvalues of matrix A.
1.2 (5%) Find the corresponding eigenvectors of the eigenvalues.
1.3 (5%) Find the matrix P that diagonalizes A.

2. (25%)

2.1 (5%) An $n \times n$ complex matrix A is unitary if and only if $\overline{A}^{-1} = A^t$ where $\overline{A}$ and $A^t$ are respectively the conjugate and transpose of matrix $A$. Show that $A=\begin{pmatrix} j/\sqrt{2} & 1/\sqrt{2} \\ -j/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is a unitary matrix where $j=\sqrt{-1}$.

2.2 (5%) An $n \times n$ complex matrix A is Hermitian if and only if $\overline{A} = A^t$ where $\overline{A}$ and $A^t$ are respectively the conjugate and transpose of matrix $A$. Is matrix $\begin{pmatrix} 15 & 8j & 6-2j \\ -8j & 0 & -4+j \\ 6+2j & -4-j & -3 \end{pmatrix}$ Hermitan?

2.3 (5%) Let $Z$ be an $n \times 1$ complex matrix. Show that $\overline{Z}^t HZ$ is real if $H$ is an $n \times n$ Hermitian matrix.

2.4 (5%) An $n \times n$ complex matrix $A$ is skew-Hermitian if and only if $\overline{A} = -A^t$. Is matrix $\begin{pmatrix} 0 & 8j & 2j \\ 8j & 0 & 4j \\ 2j & 4j & 0 \end{pmatrix}$ skew-Hermitan?

2.5 (5%) Let $Z$ be an $n \times 1$ complex matrix. Show that $\overline{Z}^t SZ$ is zero or pure-imaginary if $S$ is an $n \times n$ skew-Hermitian matrix.

3 (10%) Calculate the following matrix-vector multiplication:

$$\begin{pmatrix} \cos(\pi/16) & \sin(\pi/16) \\ -\sin(\pi/16) & \cos(\pi/16) \end{pmatrix} \begin{pmatrix} \cos(9\pi/16) & \sin(9\pi/16) \\ -\sin(9\pi/16) & \cos(9\pi/16) \end{pmatrix} \begin{pmatrix} \cos(2\pi/16) & -\sin(2\pi/16) \\ \sin(2\pi/16) & \cos(2\pi/16) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Hint: $\begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ is a clockwise rotation of vector $\begin{pmatrix} x \\ y \end{pmatrix}$ by an angle of $\theta$.

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科目：工程數學【資工系碩士班乙組】

4 (15%)

The Fourier transform $X(j\omega)$ of $x(t)$ is defined as $X(j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\omega t}dt$. $x(t)$ is called the inverse Fourier transform of $X(j\omega)$ and is given by $x(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\omega)e^{j\omega t}d\omega$.

4.1 (5%) Prove the Parseval's Relation $\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$.

4.2 (5%) Prove that $y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau \Rightarrow Y(j\omega)=H(j\omega)X(j\omega)$ where $Y(j\omega),H(j\omega),X(j\omega)$ are the Fourier transforms of $y(t),h(t),x(t)$ respectively.

4.3 (5%) For an ideal lowpass filter whose frequency response is given by

$$H(j\omega)=\begin{cases}1 & |\omega|<\omega_c\\ 0 & |\omega|>\omega_c\end{cases}$$

Show that the corresponding impulse response, the inverse Fourier transform $H(j\omega)$, is $h(t)=\frac{\sin(\omega_c t)}{\pi t}$. Draw the function of the impulse response $h(t)=\frac{\sin(\omega_c t)}{\pi t}$.

5 (10%)

The Z-transform $X(z)$ of a discrete-time signal $x(n)$ is defined as $X(z)=\sum_{n=-\infty}^{\infty}x(n)z^{-n}$.

5.1 (5%) Find the Z-transform of the function $a\times u(n)$ with $|a|<1$ where $u(n)$ is the step function defined as $u(n)=\begin{cases}1 & n=0,1,2,\cdots\\ 0 & \text{otherwise}\end{cases}$

5.2 (5%) Consider the Z-transform $X(z)=4z^2+2+3z^{-1},\quad 0<|z|<\infty$. What is the corresponding discrete-time sequence $x(n)$?

6 (25%)

The Laplace transform $X(s)$ of a function $x(t)$ is defined as $X(s)=\int_{0}^{+\infty}x(t)e^{-st}dt$.

6.1 (5%) Find the Laplace transform of the function $x(t)=1$.

6.2 (5%) Find the Laplace transform of the function $x(t)=e^{at}$.

6.3 (5%) Find the inverse Laplace transform of $X(s)=\frac{2s-1}{(s-1)(s^2+4s+3)}$.

6.4 (10%) Solve the following differential equation

$$y''(t)+4y'(t)+3y(t)=e^t,\quad y(0)=0,\ y'(0)=2$$