

一、單複選

1. (3 分) 在多層式 (Multi-Tiers) 平台裡，客戶端電腦存放的程式為 (A) Interrupt 部份，(B) Presentation 部份，(C) Logic 部份，(D) Data 部份，(E) Queue 部份。
2. (3 分) 等價關係 (Equivalence Relation) 必須滿足下列何項性質？(A) Reflexive，(B) Exclusive，(C) Symmetric，(D) Transitive，(E) Additive。
3. (3 分) Syntax analyzer 的輸入資料為何？(A) String，(B) Intermediate Code，(C) Token，(D) Machine Code，(E) Loader。
4. (3 分) A pure procedure (A) does not modify itself，(B) is also known as a recursive procedure，(C) is sharable，(D) has no goto statement，(E) must be written in assembly language。
5. (3 分) 在 Web Programming 裡，下面那些屬於 Client-Side Scripting？(A) VB Script，(B) Java Applet，(C) Java Script，(D) ASP.Net，(E) Flash。
6. (3 分) 下面那些屬於多形 (Polymorphism)？(A) Overloading 多形，(B) Coercion 多形，(C) Composition 多形，(D) Inclusion 多形，(E) Parametric 多形。
7. (3 分) 下面那些屬於 OSI protocol 的 Layer？(A) Duplex Layer，(B) Data Link Layer，(C) Fiber Layer，(D) Network Layer，(E) Transport Layer。
8. (3 分) 下面那些屬於 Unified Modeling Language (UML) 描述軟體行為 (Software Behavior) 的 Diagram？(A) Class Diagram，(B) Sequence Diagram，(C) Component Diagram，(D) Collaboration Diagram，(E) Use Case Diagram。

二、問答選

1. (6 分) 說明 Recursion 和 Loop 兩者之間相同與相異之處。
2. (6 分) 什麼是結構化程式 (Structured Programming)？
3. (7 分) 什麼是結構化系統開發方法 (Structured System Development Methodology)？它有何缺點？
4. (7 分) 請用數學的觀念來解釋資料 (Data) 與軟體 (Software) 兩者之間的關係。
5. (10 分) Given a doubly linked circular list L with length  $> 2$ , let p be a pointer pointing to a node in the list. Please write a sequence of program codes to delete the node pointed by p from the list.
6. (10 分) Most of computer systems nowadays use RAID (Redundant Array of Independent Disk) to improve their reliability and performance of the storage system. Please describe the terms mirroring RAID and parity RAID. What are the advantages and disadvantages of mirroring RAID and parity RAID?

7. (10 分) Consider a computer system with four resources of the same type that are shared by three processes, each of process needs at most two resources. Please prove that the system is deadlock free.
  
8. (20 分) Design an algorithm to sort three numbers. You should make it to have the least number of comparisons in the average case. How many comparisons are required in the best, worst and averages cases in your algorithm, respectively?

1. (25分)

企業電子化是企業未來經營模式的主要趨勢，請問

(a) 何謂企業電子化 (E-Business, EB)？

(b) EB 在資訊科技 (IT) 上的運用與傳統的 MIS 相比較，有那些重要的不同之處？

2. (25分)

「知識管理」是目前企業尋求競爭優勢提昇與經營模式轉型的最重要策略，然而一個企業知識管理的成功需要企業具備許多各方面優秀的基礎能力來支援。請問，你認為支援知識管理成功最重要的基礎能力 (或條件) 有那幾種？請分別說明。

3. (a) There exist many types of data constraints in real-world database applications. The entity relationship (ER) data model provides some modeling constructs to represent data constraints (e.g., uniqueness constraint, cardinalities of association relationships, disjointness and completeness constraints of specialization relationships, etc). However, not all of the constraints existing in a real-world application can be represented in the entity relationship model. Please identify and elaborate **three** examples of data constraints that cannot be represented in the ER model. (15分)

(b) The ER model can capture more data semantics/constraints than the relational data model can. Thus, when transforming an ER schema into a relational schema, some data semantics/constraints represented in the ER schema may be lost. Please identify **two** examples to illustrate the semantic lost problem. (10分)

4. Time-critical services and location-sensitive services represent two major types of mobile commerce (or *m-commerce*) applications. An example of time-critical service is SMS (short message service) based notifications or alerts (e.g., airline flight schedule changes, stock price alerts) for providing the time-critical value to users. On the other hand, location-sensitive services rely on the location information of moving targets for delivering “relevant” and “appropriate” services. For instance, location-sensitive services can help mobile users locate nearby restaurants and shops carrying products with certain specifications.

(a) What are the major design focuses of each of these two types of m-commerce systems? Please explain your answers in details. (12分)

(b) Assume that you want to apply a data mining technique to deliver a more “intelligent” location-sensitive service. Please give an example of an “intelligent” location-sensitive service. How can a data mining technique be used to support the example you described in (a)? (13分)

1. (15分) 某研究單位針對大學畢業生的第一份工作予以統計分析，以下資料是商學院的男性和女性畢業生在服務業的分類百分比：

	資訊業	餐旅業	百貨流通業	金融服務業	其他	總共
男性	30	15	15	30	10	100
女性	15	25	30	25	5	100

檢定性別與服務業別是否獨立 ( $\alpha = .05$ ) ?

2. (15分) 「陽光電子報」的行銷經理假設其讀者的平均年齡是 20 歲：

$$H_0: \mu = 20$$

$$H_1: \mu \neq 20$$

- 讀者年齡可視為常態分配。已知  $\sigma = 4$ ，令樣本數為 25。在  $\alpha = .05$  的情況下，當母體實際的期望值為 22 ( $\mu = 22$  歲) 時，接受  $H_0$  的機率是多少？
- $\mu = 22$  歲時，檢定強度是多少？

3. (20分) 就下列資料，請做 ANOVA 檢定 ( $\alpha = .05$ ；答案必須包括：假設、檢定規則、ANOVA 表格、結論)。

數據資料	A 組	B 組	C 組
1	50	53	44
2	44	58	39
3	47	63	48
4	51	54	41
樣本平均值	48	57	43
樣本變異數	10	20.67	15.33

4. (10分) 令  $\theta$  為指數分配的期望值，證明  $\bar{X} = \sum_{i=1}^n X_i / n$  為  $\theta$  的不偏估計式 (unbiased estimator)。

5. 請回答下列問題 (每空格 2 分)

- 一個隨機實驗 (random experiment) 是某生接受一個 10 個题目的考試；則隨機變數是 \_\_\_\_\_，此隨機變數的樣本空間是 \_\_\_\_\_。
- 在假設檢定中，所謂「型一錯誤」的定義是 \_\_\_\_\_，其機率用什麼數學符號作代表？ \_\_\_\_\_。
- 在個數為  $n$  的抽樣架構中，每一個體都有一樣的機率被抽驗，這種抽樣方法稱為 \_\_\_\_\_。

6. In an environmental study of 1072 men, a multiple regression was calculated to show how lung function was related to several factors, including some hazardous occupations :

$$\text{AIRCAP} = 4500 - 39X_1 - 90X_2 - 350X_3 - 380X_4 - 180X_5$$

(SE)                      (1.8)    (2.2)    (46)    (53)    (54)

Where

AIRCAP = air capacity (milliliters) that the worker can expire in one second.

$X_1$  = age (years)

$X_2$  = amount of current smoking (cigarettes per day)

$X_3$  = 1 if subject is a chemical worker, 0 if not

$X_4$  = 1 if subject is a farm worker, 0 if not

$X_5$  = 1 if subject is a firefighter, 0 if not

A fourth occupation, physician, served as the reference group, and so did not need a dummy. Assuming these 1072 people were a random sample.

- a. Calculate the 95% confidence interval for each coefficient. (5 分)

Fill in the brackets for the next 5 questions.

- b. Other things being equal, chemical workers on average have AIRCAP values that are (      ) milliliters lower than physicians. (2 分)
- c. Other things being equal, chemical workers on average have AIRCAP values that are (      ) milliliters higher than farmworkers. (2 分)
- d. Other things being equal, on average a man who is 1 year older has an AIRCAP value that is (      ) milliliters lower. (2 分)
- e. Other things being equal, on average a man who smokes one pack (20 cigarettes) day has an AIRCAP value that is (      ) milliliters lower. (2 分)
- f. As far as AIRCAP is concerned, we estimate that smoking one pack a day is roughly equivalent to aging (      ) years. (2 分)

Based on the results in f, answer the following question.

- g. State your reasons that the estimate in f may be biased. (5 分)

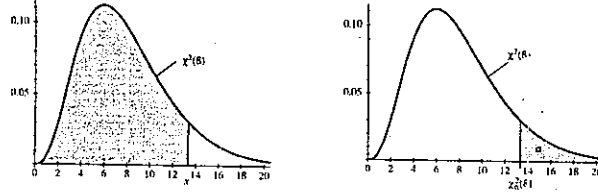
7. Answer True or False ; If False, correct it.

- a. If the alternative hypothesis is two-sided, then the p-value, classical test, and confidence interval should two-sided too. (5 分)
- b. To decide whether the probability  $\pi$  of a die coming up ace is fair, suppose we are testing the hypothesis. (5 分)

$$H_0 : \pi = 1/6 \quad \text{against} \quad H_A : \pi < 1/6$$

Then we should use a two-sided test, rejecting  $H_0$  when  $\pi$  turns out to be large.

The Chi-Square Distribution



$$P(X \leq x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2} dw$$

r	P(X ≤ x)							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
	$\chi_{0.99}^2(r)$	$\chi_{0.975}^2(r)$	$\chi_{0.95}^2(r)$	$\chi_{0.90}^2(r)$	$\chi_{0.10}^2(r)$	$\chi_{0.05}^2(r)$	$\chi_{0.025}^2(r)$	$\chi_{0.01}^2(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09
6	0.872	1.237	1.635	2.204	10.64	12.59	14.45	16.81
7	1.239	1.690	2.167	2.833	12.02	14.07	16.01	18.48
8	1.646	2.180	2.733	3.490	13.36	15.51	17.54	20.09
9	2.088	2.700	3.325	4.168	14.68	16.92	19.02	21.67
10	2.558	3.247	3.940	4.865	15.99	18.31	20.48	23.21
11	3.053	3.816	4.575	5.578	17.28	19.68	21.92	24.72
12	3.571	4.404	5.226	6.304	18.55	21.03	23.34	26.22
13	4.107	5.009	5.892	7.042	19.81	22.36	24.74	27.69
14	4.660	5.629	6.571	7.790	21.06	23.68	26.12	29.14
15	5.229	6.262	7.261	8.547	22.31	25.00	27.49	30.58

$$P(F \leq f) = \int_0^f \frac{\Gamma((r_1 + r_2)/2) \Gamma(r_1/2) r_1^{r_1/2} w^{r_1/2-1}}{\Gamma(r_1/2) \Gamma(r_2/2) (1 + r_1 w/r_2)^{(r_1+r_2)/2}} dw$$

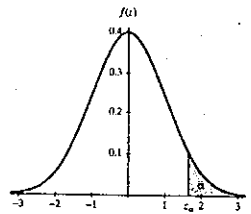
P(F ≤ f)	Den. d.f. r <sub>2</sub>	Numerator Degrees of Freedom, r <sub>1</sub>									
		1	2	3	4	5	6	7	8	9	10
0.95	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9
0.975		647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28	968.63
0.99		4052	4999.5	5403	5625	5764	5859	5928	5981	6022	6056
0.95	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40
0.975		38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40
0.99		98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40
0.95	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
0.975		17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42
0.99		34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23
0.95	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
0.975		12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84
0.99		21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55
0.95	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
0.975		10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62
0.99		16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05
0.95	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
0.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46
0.99		13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87
0.95	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
0.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76
0.99		12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62
0.95	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
0.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30
0.99		11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81
0.95	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
0.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96
0.99		10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26
0.95	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
0.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72
0.99		10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85

# 國立中山大學九十四學年度碩士班招生考試試題

科目：統計學【資管系碩士班甲組】

共 4 頁 第 4 頁

The Normal Distribution



$$P(Z > z_\alpha) = \alpha$$

$$P(Z > z) = 1 - \Phi(z) = \Phi(-z)$$

$z_\alpha$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014

## 一、單複選

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2. (3 分) 等價關係 (Equivalence Relation) 必須滿足下列何項性質？(A) Reflexive，(B) Exclusive，(C) Symmetric，(D) Transitive，(E) Additive。
3. (3 分) 下面那些屬於 Operating System 管理 Memory 的方法？(A) Accumulator，(B) Paging，(C) Segmentation，(D) Backus Naur Form，(E) SPOOLING。
4. (3 分) A pure procedure (A) does not modify itself，(B) is also known as a recursive procedure，(C) is sharable，(D) has no goto statement，(E) must be written in assembly language。
5. (3 分) 在 Web Programming 裡，下面那些屬於 Client-Side Scripting？(A) VB Script，(B) Java Applet，(C) Java Script，(D) ASP.Net，(E) Flash。
6. (3 分) 下面那些屬於多形 (Polymorphism)？(A) Overloading 多形，(B) Coercion 多形，(C) Composition 多形，(D) Inclusion 多形，(E) Parametric 多形。
7. (3 分) 下面那些屬於 Operating System 管理的資源？(A) Processor，(B) Language，(C) Stack，(D) Files，(E) EBCDIC。
8. (3 分) 下面那些屬於 Unified Modeling Language (UML) 描述軟體行為 (Software Behavior) 的 Diagram？(A) Class Diagram，(B) Sequence Diagram，(C) Component Diagram，(D) Collaboration Diagram，(E) Use Case Diagram。

## 二、問答選

1. (6 分) 說明 Light Weight Process (LWP) 和 Heavy Weight Process (HWP) 相同與相異之處。
2. (6 分) 什麼是結構化程式 (Structured Programming)？
3. (7 分) Microsoft 公司的 Win95 是一個真正的 Multitasking 作業系統。它的前身 Windows 3.0 卻被稱之為一個虛假的 Multitasking 作業系統，Why？
4. (7 分) 請用數學的觀念來解釋資料 (Data) 與軟體 (Software) 兩者之間的關係。
5. (10 分) Describe the mechanism used in Gigabit Ethernet for maintaining 802.3 minimum and maximum frame sizes with meaningful cabling distances and how to solve the problem of low bandwidth utilization issue?
6. (10 分) Describe the five performance criteria used for comparing CPU-scheduling algorithms.



7. (10 分) Please trace the following C program and give your answer for the result returned by DaVinciCode(3) in the main program.

```
Int DaVinchCode(x)
int x;
{
    Return FascinatingStory(x, 5);
}
int FascinatingStory(x, y)
int x, y;
{
    if (x == 0) return y;
    else {
        return(FascinatingStory(x-1, y) + FascinatingStory(x-1, y));
    }
}
```

8. (20 分) Please answer the following two questions about critical session problem in operating system.
- What are the requirements that a correct solution to the critical session problem must satisfy?
  - Design a correct program to solve the critical section problem for two processes. Please justify that your solution is correct.

1. (15%)

Consider the following definition about *Strange Graph*:

A strange graph is a tuple  $(V, E, f)$ , where  $V$  is a finite set,  $E$  a set of reflexive and asymmetric binary relation on  $V$ , and  $f: V \rightarrow E$  an injection (or called one-to-one function).

In the following, which are strange graphs and which are not? For each non-strange graph, describe which parts of the above definition are violated.

A.

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_3, v_4)\}$$

$$f = \{v_1 \rightarrow (v_1, v_2), v_2 \rightarrow (v_1, v_3), v_3 \rightarrow (v_2, v_3), v_4 \rightarrow (v_3, v_4)\}$$

B.

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{(v_1, v_1), (v_2, v_2), (v_3, v_3), (v_4, v_4)\}$$

$$f = \{v_1 \rightarrow (v_1, v_1), v_2 \rightarrow (v_2, v_2), v_3 \rightarrow (v_3, v_3), v_4 \rightarrow (v_4, v_4)\}$$

C.

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{(v_1, v_1), (v_2, v_2), (v_3, v_3), (v_2, v_3)\}$$

$$f = \{v_1 \rightarrow (v_1, v_1), v_2 \rightarrow (v_2, v_2), v_3 \rightarrow (v_3, v_3)\}$$

D.

$$V = \{v_1, v_2, v_3, v_4\}$$

$$E = \{(v_1, v_1), (v_2, v_3), (v_3, v_4), (v_3, v_3)\}$$

$$f = \{v_1 \rightarrow (v_1, v_1), v_2 \rightarrow (v_2, v_3), v_3 \rightarrow (v_3, v_4)\}$$

E.

$$V = \{v_1, v_2, v_3\}$$

$$E = \{(v_1, v_1), (v_2, v_2), (v_3, v_3), (v_2, v_3), (v_3, v_2)\}$$

$$f = \{v_1 \rightarrow (v_1, v_1), v_2 \rightarrow (v_2, v_2), v_3 \rightarrow (v_2, v_3)\}$$

2. (20%)

Consider a new rule for evaluating an expression. The new rule regards / and + to have higher priorities than \* and -, and the evaluation of operators of the same priority will proceed right to left. For example, accordingly to the new rule, the expression  $A * B - C - D + E / F$  will be evaluated as

$$(A * ((B - (C - (D + E))) / F))$$

A. What are the results of evaluating the following two expressions using the new rule? (5%)

$$1 - 2 - 4 + 3 - 4 - 5 * 9 / 3 / 3$$

$$5 * 2 + 100 - 20 - 70 / 15$$

B. Write the pseudo code that outputs the postfix form of an infix expression  $e$ . based on the new rule. Ensure to describe the data structures you use in your algorithm (15%)

3. (15%)

Write an algorithm to list all the identifiers in a hash table in lexicographical order. Assume that the hash function  $h$  is  $h(x) = \text{first character of } x$ . For example,  $h('a')=0$ ,  $h('b')=1$ ,  $h('c')=1$ , ...,  $h('z')=25$ . In the hash table, identifiers are assumed to consist of only small English letters, and linear probing is used.

4. (10%)

A permutation of a set  $X$  is a mapping from  $X$  into itself. Let  $S_n$  denote the set of permutations of  $[n] = \{1, 2, \dots, n\}$  ( $n \in \mathbb{N}$ ). Prove  $|S_n| = n!$ .

5. (10%)

Consider bitwise logic operations. Suppose  $p$ ,  $q$  and  $x$  are 8 bit operands and we have:  $q \vee (p \wedge x) = 00111001$ ,  $q \wedge (p \vee x) = 00011001$ ,  $q \wedge p \wedge x = 00000001$ ,  $q \vee p \vee x = 10111111$ . If we know  $p = 00101001$  and  $q = 00011001$ , then can we determine  $x$ ? Can we determine it uniquely?

6. (10%)

Let  $A$  and  $B$  be countable sets and  $C$  an uncountable set. In each case show that if (necessarily) the mentioned set is countable or not?

- (a)  $A \cup B$
- (b)  $A \cap B$
- (c)  $A - B$
- (d)  $X$  (where  $X \subseteq A$ )
- (e)  $A \cup C$
- (f)  $A \cap C$
- (g)  $A - C$
- (h)  $X$  (where  $X \supseteq C$ )

7. (20%)

Recall that  $N$  denotes the set of natural numbers, and  $Z$  denotes the set of integers.

(a) Define  $P(n)$  by

$$P(n) = \forall m \in N, m < n \Rightarrow \neg(\exists k \in N, n = mk \wedge k < n)$$

Concisely, for which numbers in  $N$ , is  $P(n)$  true?

(b) Rewrite the following in a way that removes all negations ( $\neg$ ,  $\neq$ ) but remains equivalent:

$$\forall i, \neg \forall j, \neg \exists k (\neg \exists l, f(i, j) \neq g(k, l))$$

(c) Prove or disprove:  $\forall m \in Z, \exists n \in Z, m \geq n$ .

(d) Prove or disprove:  $\exists m \in Z, \forall n \in Z, m \geq n$