

國立中山大學99學年度碩士班招生考試試題

科目：工程數學【海下海物所碩士班選考】

1. (20%) Consider the ordinary differential equation (ODE): $my''(t) + cy'(t) + ky(t) = 0$.
- (a) If m, c, k are positive constants, show that all solutions of the ODE approach zero as $t \rightarrow \infty$. (10%)
- (b) If m, k are positive constants, but $c = 0$, show that all the solutions are bounded. (5%)
- (c) Describe, respectively, the physical significance of the solutions obtained in (a) and (b) in terms of mechanical vibration. (5%)

2. (20%) Let $y_1(x)$ be a solution of the homogeneous equation corresponding to the following ODE:

$$y'' + a(x)y' + b(x)y = f(x)$$

- (a) Describe the procedure for finding the second independent solution $y_2(x)$ for the homogeneous equation corresponding to the ODE, and then derive $y_2(x)$ symbolically. (10%)
- (b) Describe the procedure for finding the particular solution $y_p(x)$, and then derive $y_p(x)$ symbolically. (10%)
3. (15%) Solve the initial value problem:

$$y'' + 3y' + 2y = \delta(t - 5) + u_{10}(t); \quad y(0) = 0, \quad y'(0) = 1$$

where $\delta(t - 5)$ is the Dirac delta function that has an impulse at $t = 5$, and $u_{10}(t)$ is the step function that has a unit step jump at $t = 10$.

4. (20%) Let \mathbf{F} be a vector field defined as: $\mathbf{F}(x, y, z) = 8x \mathbf{i} - 4y^2 \mathbf{j} + 2x^2z^2 \mathbf{k}$.
- (a) Find $\nabla \cdot \mathbf{F}$ at $(1, -1, 1)$. (5%)
- (b) Find $\nabla \times \mathbf{F}$ at $(1, -1, 1)$. (5%)
- (c) Find the volume integral: $\iiint_V \nabla \cdot \mathbf{F} dV$ over the region inside the cylinder defined by the surface $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (10%)
5. (25%) The equilibrium state of heat transfer in a thin circular disk of radius a may be described by the following partial differential equation (PDE):

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0, \quad 0 < r < a, \quad -\pi < \theta < \pi$$

Let the boundary condition be given as:

$$u(a, \theta) = f(\theta),$$

solve the PDE system.

1. 求極限 $\lim_{x \rightarrow 0} \frac{(x+2)^{10} - 1024}{x}$ (5%)。

2. 計算以下的積分

(a) $\int \sin^9 x \, dx$ (5%)

(b) 利用積分證明圓的面積公式。(5%)

(c) $\int_0^1 \frac{x e^{\sqrt{x^2+1}}}{\sqrt{x^2+1}} dx$ (5%)

3. $\omega(k) = \sqrt{gk \tanh kd}$, 求 $\frac{\left(\frac{d\omega}{dk}\right)}{\left(\frac{\omega}{k}\right)}$ 。(10%)

4. (a) 以 Taylor Series 將 $\sqrt{1+x}$ 對 x 展開至 x^3 階 (5%)

(b) 利用以上結果求 $\sqrt{65}$ 的近似值並探討其誤差 (10%)

5. 在時間 t 時，一質點的位置為 $\vec{r} = 3t \vec{i} + \sinh t \vec{j} + 2t^3 \vec{k}$ ，求該質點在時間 t 時的速度與加速度。(各 5%)

6. 旋度與散度

$$\vec{F}(x, y, z) = 3xz^2\vec{i} + y\sin z\vec{j} + ye^{2z}\vec{k}, \text{ 求 } \nabla \wedge \vec{F} \text{ 與 } \nabla \cdot \vec{F}.$$

(各5%)

$$7. \text{ 設 } f(x) = \int_{3x}^{x^3-5} \frac{t}{10-\sqrt{t}} dt, \text{ 求導數 } f'(3) \text{ 之值。 (10\%)}$$

8. 假設 $x=10$ 公里處產生波動，以每小時 $\frac{x^2}{6}$ 公里的速度向 $+x$ 方向傳播，求(1)在 x 處波經過 dx 距離需要多少時間？(5%) (2)多少時間後，波會抵達座標 $x=40$ 公里處？(10%)

9. 求由點 $(2, 1, 1)$ 到平面 $x+y+z=0$ 的最短距離。(10%)

名詞解釋 (每題 5%，共 20%)

1. Siphon
2. Capillary action
3. Navier-Stokes equations
4. Flow separation

計算題 (共 80%)

1. Given a two-dimensional flow in the x - y plane, the x component of velocity is given by $u = Ax$. Determine a possible y component for incompressible flow. (20%)
2. By using the SI units, to prove the Reynolds number is dimensionless. (15%)
3. If the liquid in a syringe, Fig. 1, is to be injected steadily at a rate of $300 \text{ cm}^3/\text{min}$; and the plunger has a face area of 500 mm^2 , at what speed should be the plunger be advanced? The leakage rate past the plunger is 0.1 times the volume flow-rate out of the needle. (25%)

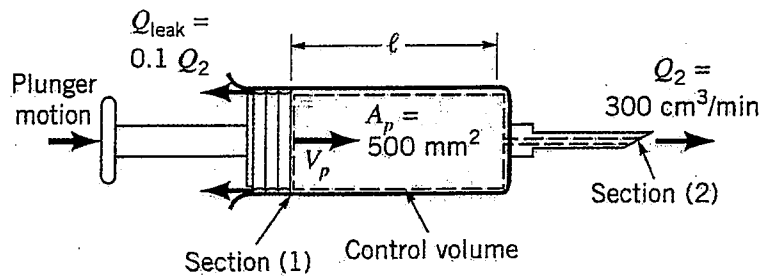


Figure 1

4. A pitot tube is inserted in an air flow to measure the flow speed, as in Fig. 2. The tube is inserted so that it points upstream into the flow and the pressure sensed by the tube is the stagnation pressure. The static pressure is measured at the same location in the flow, using a wall pressure tap. If the pressure difference is 30 mm of mercury, determine the flow speed. (20%) (Hint: SG of mercury is 13.55)

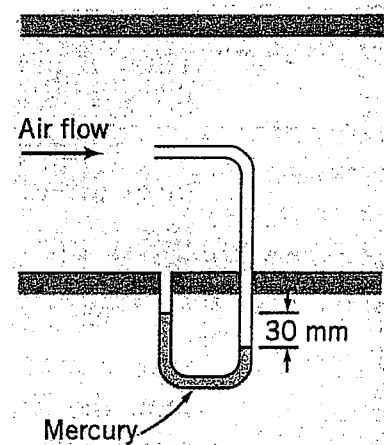


Figure 2

- (16%) 1. The voltage regulator, shown in Figure 1, is to power a car radio at $V_L = 9V$ from an automobile battery whose voltage may vary between 11 and 13.6V. The current in the radio will vary between 0 (off) to 100 mA (full volume). Calculate (a) (4%) the maximum Zener diode current, $I_Z(\max)$, (b) (4%) the maximum power dissipated in the Zener diode, $P_Z(\max)$, (c) (4%) the value of the current-limiting resistor, R_i , and (d) (4%) the maximum power dissipated in this resistor, $P_{R_i}(\max)$.
- (14%) 2. Calculate the dc voltages at each node and the dc currents through the elements in Figure 2. Assume the B-E turn-on voltage is 0.7V and $\beta = 100$ for each transistor.
- (15%) 3. For the circuit in Figure 3, assume the transistor parameters are: $\beta = 100$, $V_{BE(\text{on})} = 0.7V$, and $V_A = 100V$. Determine (a) (5%) the small-signal voltage gain A_v , (b) (5%) input resistance R_i , and (c) (5%) output resistance R_o .
- (10%) 4. For the dc circuit in Figure 4, assume the MOSFET parameters are $V_{TN} = 2V$, $k_n' = 80\mu A/V^2$, and $W/L = 4$. Calculate R_1 and R_2 such that the current in the bias resistors is approximately one-tenth of I_D , where $I_D = 0.5$ mA.
- (20%) 5. (a) (10%) Determine the corner frequencies (f_L and f_H) and bandwidth (f_{BW}) of the circuit shown in Figure 5 with parameters $R_S = 1$ k Ω , $R_p = 10$ k Ω , $C_S = 1\mu F$, and $C_p = 3$ pF. (b) (10%) Also show the Bode plot of the magnitude of the voltage transfer function for the circuit.
- (15%) 6. Consider the circuit in Figure 6. Let $Z_L = 100$ Ω , $R_1 = 10$ k Ω , $R_2 = 1$ k Ω , $R_3 = 1$ k Ω , and $R_F = 10$ k Ω . If $v_i = -5V$, determine (a) (5%) i_L , (b) (5%) v_o , and (c) (5%) i_2 .
- (10%) 7. Derive the relationship between v_o and v_i for the circuit shown in Figure 7.

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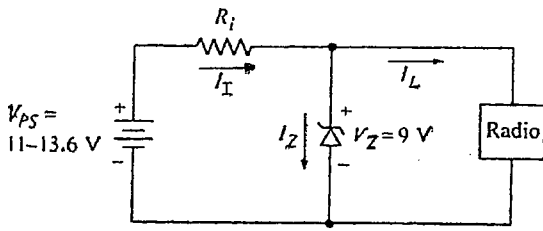


Figure 1

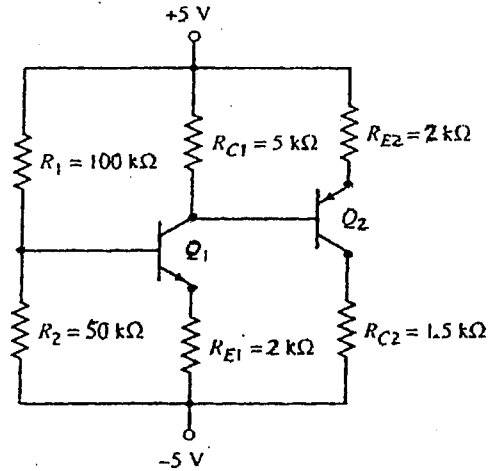


Figure 2

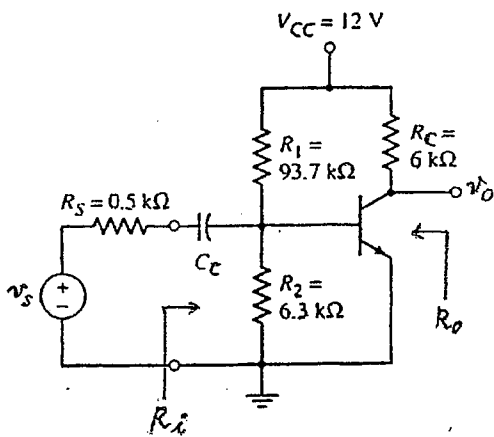


Figure 3

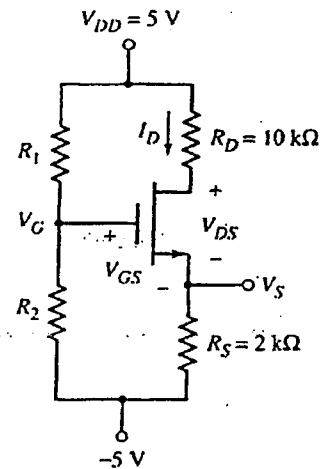


Figure 4

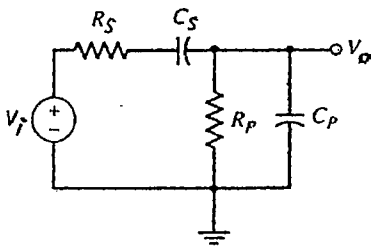


Figure 5

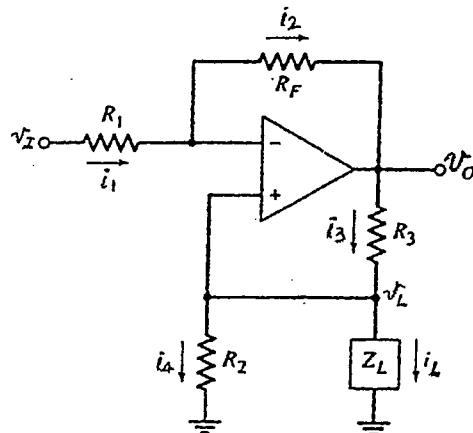


Figure 6

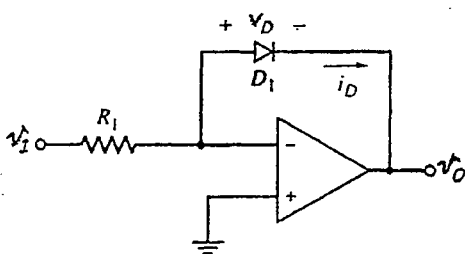


Figure 7

國立中山大學99學年度碩士班招生考試試題

科目：海洋學【海下海物所碩士班選考】

一、解釋名詞 (10 題、每題 5 分共 50 分) (請在試卷上作答否則不予計分)

1. Continental shelf
2. Hydrothermal vents
3. Red tide
4. Thermocline
5. Geostrophic current
6. Potential temperature
7. Sigma-t
8. Swell
9. Tsunami
10. SOFAR channel

二、申論題 (2 題共 50 分) (請勿在本試題紙上作答否則不予計分)

1. 請說明造成海水流動的因素。(20 分)

2. 下圖(摘自 Science 2009.12.18 期)為全球暖化研究之溫度異常(圖 A), 顯示觀測與數值模擬相似, 1985 至 2009 增溫約 0.5°C 。溫度變化可用四個指標(圖 B)解釋 76%, 包括聖嬰、火山塵、太陽週期及人類之影響。請分項依圖描述造成各變化之因素。(30 分)

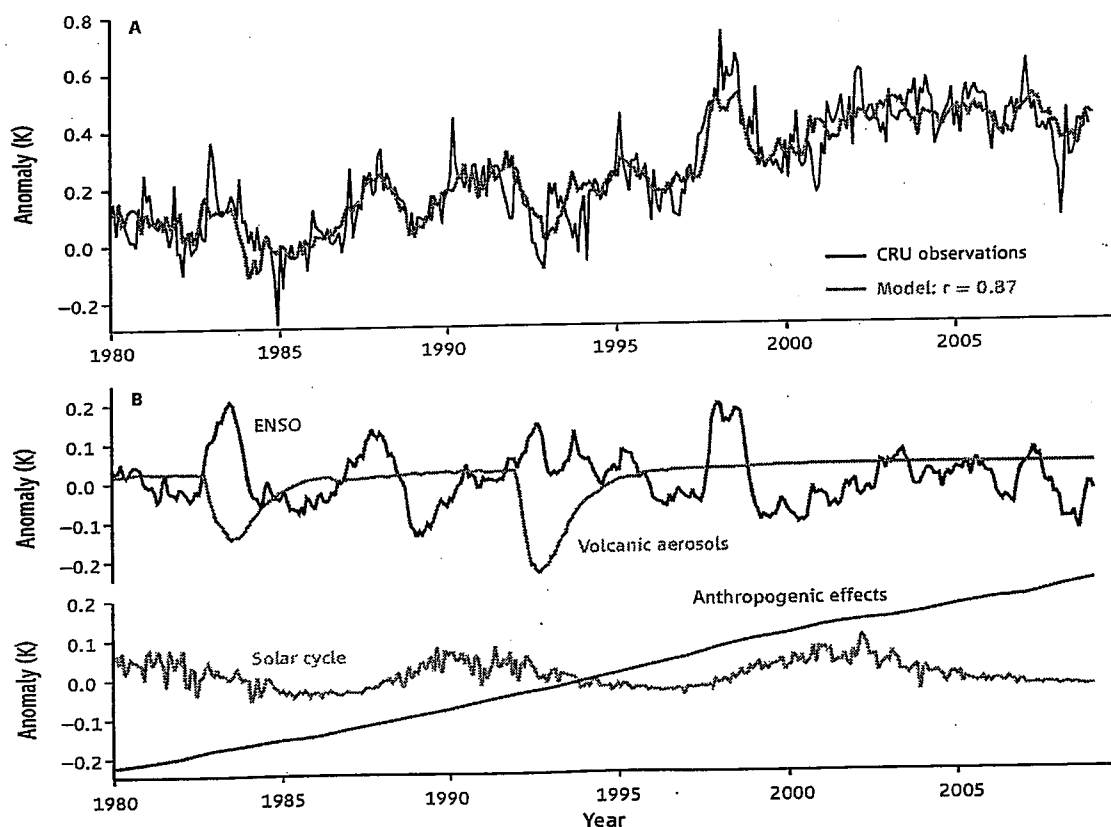


Fig. 7. Reconstructing Earth's recent climate. (A) Observed monthly mean global temperatures (black) and an empirical model (orange) that combines four different influences. (B) Individual contributions of these influences, namely El Niño–Southern Oscillation (purple), volcanic aerosols (blue), solar irradiance (green), and anthropogenic effects (red). Together the four influences explain 76% (r^2) of the variance in the global temperature observations.

說明：本試卷共五題，總分 100 分。

1. (20%) F_1 與 F_2 兩作用力的大小、位置、與方向如圖 1 所示（註： $\sqrt{2} = 1.414$ ）。

- (1) 分別將 F_1 與 F_2 以卡式向量 (Cartesian vector) 表示。(5%)
- (2) F_1 與 F_2 的合力為何？(5%)
- (3) F_1 在 F_2 方向的分力為何？(5%)
- (4) F_1 對 z 軸的作用力矩大小為何？(5%)

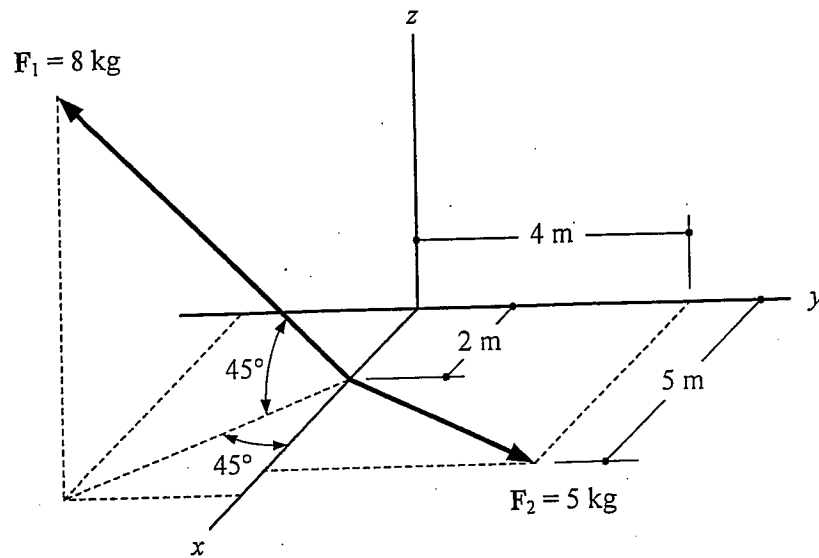


圖 1

2. (20%) 一支吊桿長 3 m，其支撐與負載狀況如圖 2 所示。其中，BC 段為繩索，支撐點 C 位於吊桿上，而吊桿與水平之夾角固定保持為 30° ，因此，若改變支撐點 C 的位置（亦即改變 θ 值），繩索張力大小也會隨之改變。在忽略吊桿重量的情況下：

- (1) 畫出吊桿的自由體圖。(5%)
- (2) 當 $\theta = 60^\circ$ 時，寫出平衡方程式。(5%)
- (3) 當 θ 等於多少時，繩索張力會達到最小（C 點必須位於吊桿上）？(10%)

[Hint: 由力矩平衡方程式著手，建立張力與 θ 的關係式。]

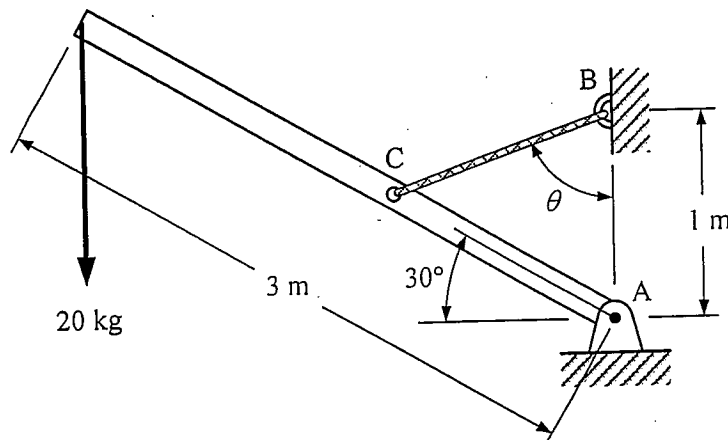


圖 2

3. (25%) 一個海洋觀測浮標由 A、B、C 三個圓柱體所組成，如圖 3 所示。假設各圓柱皆為均質物體，圓柱 A 的密度為 0.3 g/cm^3 ，圓柱 B 的密度為 1.1 g/cm^3 ，圓柱 C 的密度為 3 g/cm^3 ，各圓柱體之半徑分別為 $r_A = 50 \text{ cm}$ 、 $r_B = 10 \text{ cm}$ 、 $r_C = 20 \text{ cm}$ 。
- (1) 求出各圓柱體的形心位置座標。(5%)
 - (2) 求出海洋觀測浮標整體的重心座標。(5%)
 - (3) 令水的密度為 1.0 g/cm^3 ，浮標置於水中達到平衡之後，浮標凸出水面的高度為何？(10%)
 - (4) 觀測浮標置於水中達到平衡之後，浮標的浮心座標為何？(5%)

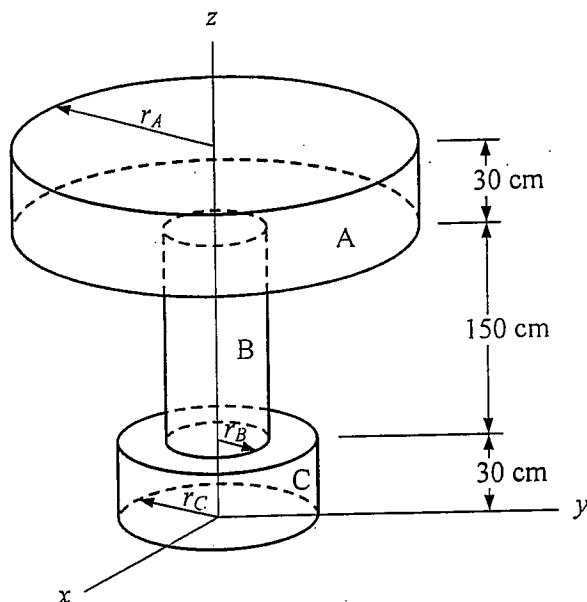


圖 3

4. (20%) 每一個鐵塊長 10 cm ，重 10 kg ，其堆疊形式如圖 4 所示。
- (1) 在圖 4(a)中，這兩個鐵塊的堆疊形式可以維持平衡而不會傾倒嗎？為什麼？(3%)
 - (2) 在圖 4(b)中，這三個鐵塊的堆疊形式可以維持平衡而不會傾倒嗎？為什麼？(7%)
 - (3) 根據圖 4(c)的堆疊方式，在不會傾倒的條件下，最上方鐵塊可以凸出最下方鐵塊的最遠距離 d 是多少？(10%)

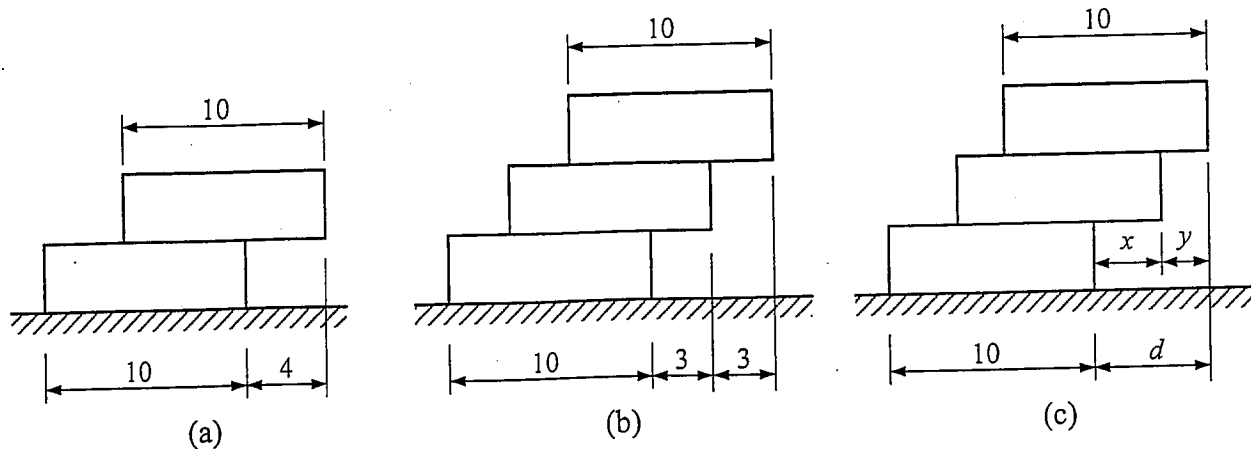


圖 4

科目：靜力學【海下海物所碩士班選考】

5. (15%) 名詞解釋：

- (1) Centroid (3%)
- (2) Conservative force (3%)
- (3) Rigid body (3%)
- (4) Statically indeterminate (3%)
- (5) Two-force member (3%)