國立中山大學九十四學年度碩士班招生考試試題

科目：工程數學【海下所碩士班】

1. (15%) (a) If the roots of the characteristic equation corresponding to a 7th-order linear non-homogeneous ordinary equation with constant coefficients are: 3, 3, 3, $2\pm3i$, $2\pm3i$, write down the solution corresponding to the homogeneous equation. [5%] (b) If the equation has non-homogeneous term as $2e^{3x} + e^{2x}\cos 3x$, construct a functional form for the particular solution, if the method of undertermined coefficients is employed. [10%]

2. (5%) If the homogeneous solutions corresponding to the *non-homogeneous* linear equation $y'' + a_1(x)y' + a_2(x)y = g(x)$ are $y_1(x)$ and $y_2(x)$, find the formal particular solution.

3. (15%) Solve the following initial-valued problem: $y'' + 3y' + 2y = \delta(t-5) + u_{10}(t)$, $y(0) = 0$, $y'(0) = 1/2$.

4. (10%) Find the directional derivative of $f(x) = 1/\sqrt{x^2+2y^2+z^2}$ at the point $P = (1, 2, 4)$ in the direction of the vector $\mathbf{a} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.

5. (10%) Solve the following system of linear equations by the method of Gaussian elimination:

$$\begin{aligned} x - y + 3z &= 2 \\ 3x - 3y + z &= -1 \\ x + y &= 3 \end{aligned}$$

6. (20%; 5% each) Locate and identify all points at which absolute maxima or minima are assumed by the following functions:

   (a) $x^{2/3} + y^{2/3}$

   (b) $(x^2 + y^2 - 2x + 2y + 2)^{1/2}$

   (c) $(y^2 - x^2)^{1/2}$

   (d) $(2x - 2y - x^2 - y^2)^{1/2}$

7. (10%) Consider the partial differential equation: $\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + k^2\phi = 0$, where ϕ is a function of r and z, and k is a constant. If the solution of ϕ is represented by: $\phi(r,z) = \int_0^\infty \psi(z)J_0(qr)q\,dq$, where J_0 is the zeroth-order Bessel function of the first kind, find the equation that $\psi(z)$ must satisfy.

8. (15%) Solve the following partial differential equation:

$$\begin{aligned} u_{xx} &= u_t,\ 0 < x < 25,\ t > 0 \\ u(0,t) &= 20,\ t > 0 \\ u(25,t) &= 40,\ t > 0 \\ u(x,0) &= 45 - 2x,\ 0 < x < 25 \end{aligned}$$

科目：流體力學【海下所碩士班選考】

1. (40%; 5% each) *Define* and *explain* the following terms:
   - (a) Lagrangian flow description
   - (b) Euler number
   - (c) Streamline
   - (d) Doppler effect
   - (e) Buckingham Pi theorem
   - (f) Newtonian fluid (Give an expression in relation to shear stress.)
   - (g) Cavitation phenomenon (Explain in what situation cavitation may occur, and state its significance.)
   - (h) Sound speed (Give definition in terms of pressure and density, and state its significance in relation to compressibility of a fluid.)

2. (10%) A smooth flat plate with length of 6 m and width of 4 m is placed in water with an upstream velocity of 0.5 m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume a laminar boundary layer.

3. (10%) A submarine moves through the seawater (specific gravity = 1.03) at a depth of 50 m with velocity 5.0 m/s. Determine the pressure at the stagnation point.

4. (10%) An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1 m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?

5. (10%) Assume that there is a distributive source generating $\dot{q}(x, y, z, t)$ amount of mass per unit time and per unit volume in a general fluid-flow field, derive the differential equation that governs the conservation of mass, in terms of fluid density ρ and fluid velocity $\mathbf{v} = (u, v, w)$.

6. (20%) Consider a steady oscillation of a plane below an incompressible *viscous* fluid with kinematic viscosity ν. If the plane oscillates in the $\pm x$-direction with a harmonic motion $U(t) = U_0 \cos \omega t$, where U_0 is a constant and ω is the angular frequency, and assume that the unsteady flow is one-dimensional $u(y, t)$, with y pointing upward.
   - (a) Make necessary assumptions and set up the governing equation and boundary conditions of the problem. [Use fluid properties such as ν, ρ, μ, etc. as needed] (10%)
   - (b) If steady oscillation is of interest, find the velocity distribution $u(y, t)$ of the fluid flow. (10%)

國立中山大學九十四學年度碩士班招生考試試題

科目：應用力學【海下所碩士班選考】

說明：本試卷共四題，每題 25 分，總分 100 分。

1. 如圖 1 所示，一大砲以仰角 θ 發射一顆砲彈，砲彈的出口速率 v 爲 400m/s，請問：
   (a) 砲彈可以擊中最遠的目標物距離 D 爲何？ (10%)
   (b) 若砲彈正好可以打中距離 D =12000m 外的目標物，試求出 θ 以及砲彈的飛行時間。(15%)

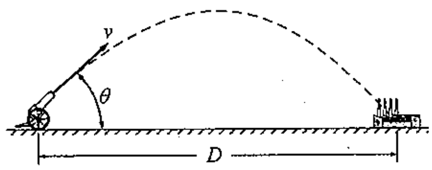

圖 1

2. 一根 1m 長的桿子利用兩條各 1m 長的繩子懸吊於 O 點，如圖 2 所示。若忽略桿子與繩子的重量，當桿子兩端各懸掛 2kg 與 5kg 的重物時，求出平衡後的傾斜角 θ。(25%)

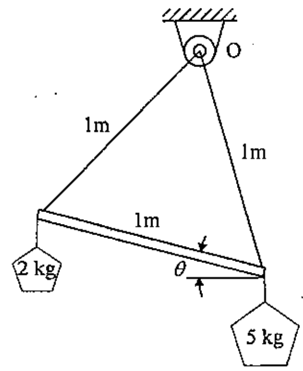

圖 2

3. 有三個方塊，A、B 及 C，質量分別爲 1Kg、2Kg 及 3Kg， A、B 及 B、C 之間距皆爲 5m，如圖 3 所示。A 以 5m/sec 的速度向右撞上靜止的 B 及 C。A、B 之間彈性碰撞係數爲 s=0.8，B、C 之間 s=0；第一段（AC 之間）爲光滑面，第二段爲粗糙面，摩擦係數爲 0.25。試問：
   (a) A、B 碰撞後速度各爲多少？
   (b) C 進入粗糙面後滑行多遠後停止？
   (c) C 靜止時，A 及 B 的位置（可以以碰撞前 C 之位置爲準描述之；不計方塊之體積）。

圖 3

4. 某項海洋實驗需要佈放兩個串聯的儀器（A 與 B）至海底測量海洋參數。實驗完成後，再以音響釋放儀（P）命令與錨定之重物（W）分離後利用浮球上升再予以回收儀器（A、B 及 P）。爲了維護資料品質，串列錨定時需盡量垂直海床，因此串列中之張力必須維持至少 10Kgw。上升回收時整個串列必須隨時維持至少 10Kgw 的浮力。整個串列如圖 4 所示，各項儀器或裝置之規格如下：儀器 A 質量 20Kg，體積 15 公升；儀器 B 質量 15Kg，體積 5 公升；音響釋放儀 P 質量 20Kg，體積 5 公升；配重每顆質量 10Kg，體積 2 公升；浮球每顆質量 2Kg，體積 10 公升。海水密度以 1000Kg/m^3 計，請問至少各需幾顆浮球及幾顆配重才能完成實驗。（注意：串列用之繩索其質量體積均不計）

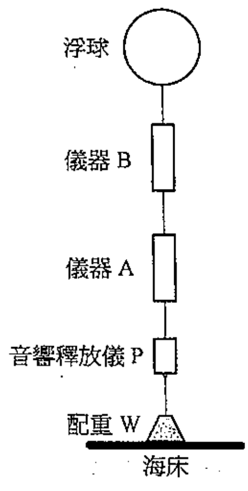

圖 4

科目：電子學【海下所碩士班選考】

1. Figure 1 shows an op amp connected in the inverting configuration. The op amp has an open-loop gain $\mu = 10^4$, a differential input resistance $R_{id} = 100\mathrm{k}\Omega$, and an output resistance $r_o = 1\mathrm{k}\Omega$. Use the feedback method to find (a) the voltage gain V_o/V_s, (b) the input resistance R_{in}, and (c) the output resistance R_{out}. 20%

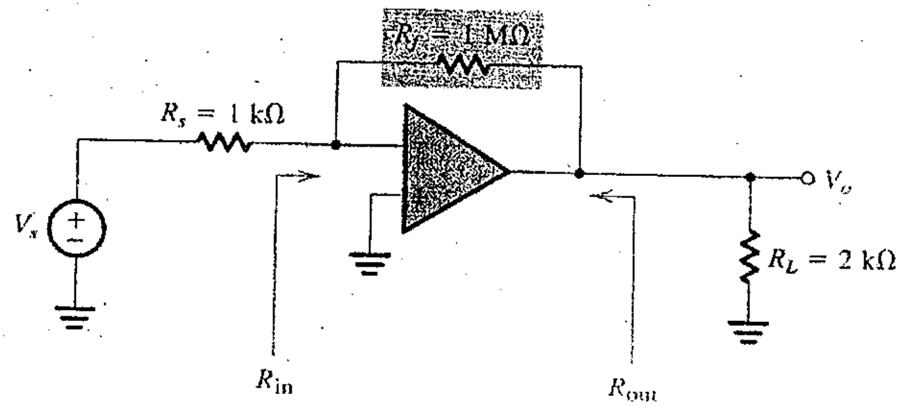

Figure 1.

2. (a) Using a simple (r_π and g_m) model for each of the two transistors Q_{18} and Q_{19} in Figure 2.a, find the small-signal resistance between A and A'. Where $I_{C18} = 165\ \mu A$ and $I_{C19} = 16\ \mu A$. 10%

(b) Figure 2.b shows the circuit for determine the 741 op-amp output resistance when v_O is positive and Q_{14} is conducting most of the current. Using the resistance of the Q_{18} and Q_{19} network calculated in 2.(a) and neglecting the large output resistance of Q_{13A}, find R_o when Q_{14} is sourcing an output current of 5mA. 10%

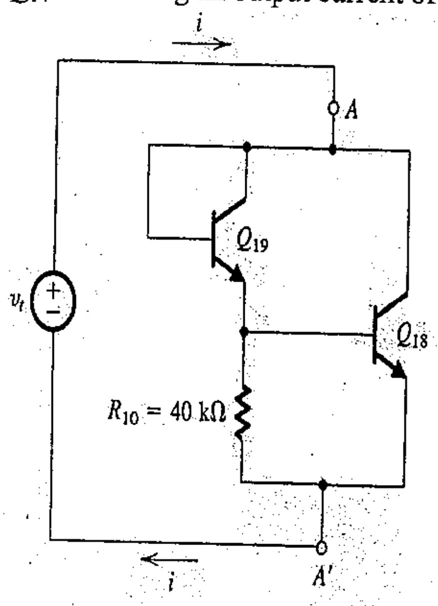

Figure 2.a

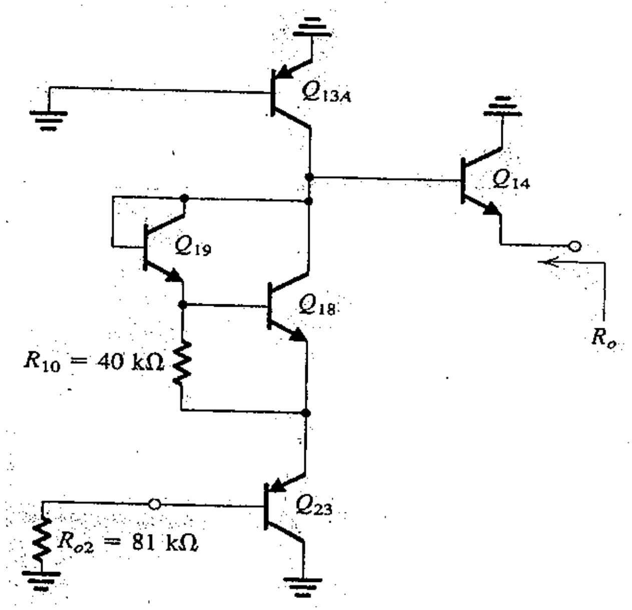

Figure 2.b

3. For the circuit shown in Figure 3, (a) break the feedback loop at X and find the loop gain βA, then (b) find the frequency of oscillation f_0 and (c) the minimum required value of R_f for oscillations to start in the circuit. 20%

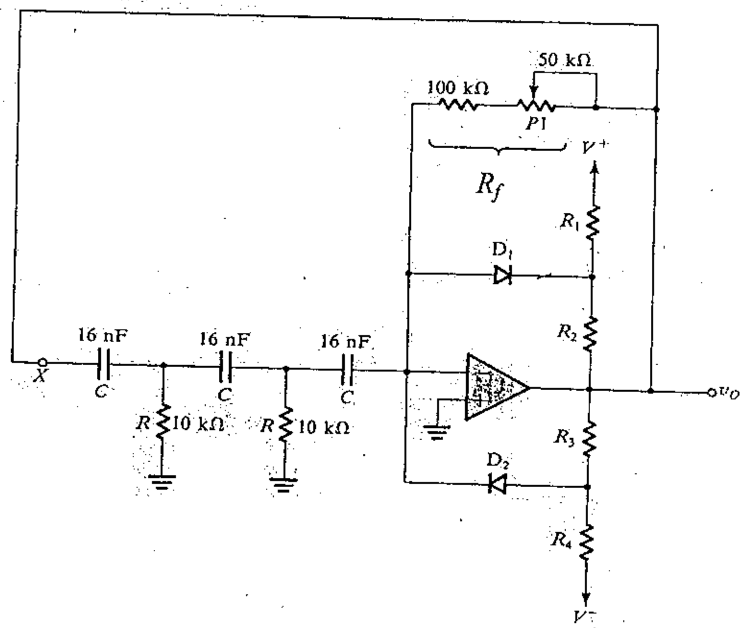

Figure 3.

4. In Figure 4 an inverter fabricated in a 0.12μm CMOS technology uses the minimum possible channel lengths (i.e. $L_n = L_p = 120$nm). (a) If $W_n = 180$nm, find the value of W_p that would result in Q_N and Q_P being matched. (b) For this technology, $k'_n = 160\mu A/V^2$ and $k'_p = 54\mu A/V^2$ with the supply voltage $V_{DD} = 1.5$ V and the threshold voltage $V_{tn} = 0.5$V. Calculate the value of the output resistance of the inverter when $v_O = V_{OL}$ ("Low" level at the output). 20%

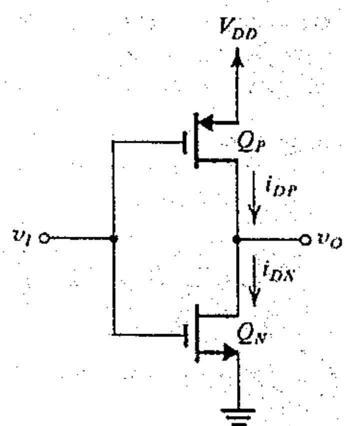

Figure 4.

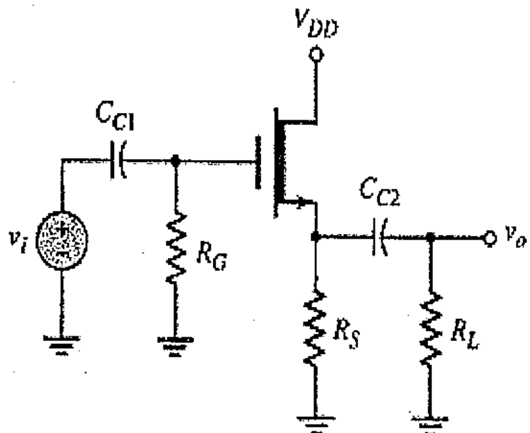

Figure 5.

5. Consider the source-follower circuit shown in Figure 5. The most negative output signal voltage occurs when the transistor just cuts off. Show that (a) this output voltage $v_o(min)$ and (b) the corresponding input voltage $v_i(min)$ with respect to I_{DQ}, g_m, R_S and R_L. 20%