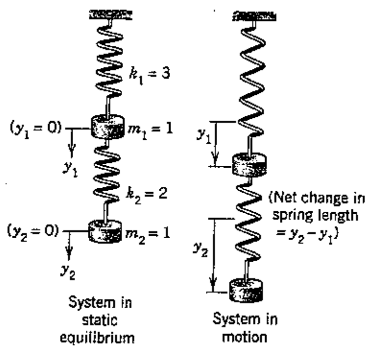
Applied Mathematics (Entrance Examination of Physics Department)

1. A particle moves so that its position vector is given by $\vec{r} = \cos\omega t\vec{i} + \sin\omega t\vec{j}$ where ω is a constant. Show that (a) the velocity **v** of the particle is perpendicular to **r**, (b) the acceleration ***a*** *is directed toward the origin and has magnitude proportional to the distance from the origin, (c)* $\vec{r} \times \vec{v} =$ *a constant vector. (10%)*

2. (a) *Show that* $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ *is a conservative force field. (b) Find the scalar potential. (c) Find the work done in moving an object in this field from (1,-2,1) to (3,1,4). (10%)*

3. *Solve the differential equation* $3y'' + 10y' + 3y = x^2 + \sin x$ *(15%)*

4. *(a) Model the undamped mechanical system in the following figure. (b) Solve the system of the two second-order differential equations directed as obtained. (15%)*



5. *Solve the differential equation by the Laplace transformtion* $y'' + 3y' + 2y = r(t)$
   $(a) r(t) = 0$
   $(b) r(t) = 1, if\ 1 < t < 2, and\ 0\ otherwise$ *(15%)*

6. *Solve the one-dimensional wave equation* $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, *the two boundary conditions are* $u(0,t) = 0, u(L,t) = 0$ *for all t, and initial conditions*

$$u(x,0) = \begin{cases} \frac{2k}{L}x, if\ 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x), if\ \frac{L}{2} < x < L \end{cases}$$ *(25%)*

$$\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$$

7. *Evaluate the following integration around the circle counterclockwise*

$\oint \frac{\tan z}{z^2 - 1} dz$, $(a) |z| = 3/2$, $(b) |z| = 1/2$ *(10%)*

國立中山大學九十一學年度碩士班招生考試試題

科目：電磁學 【物理學系碩士班】

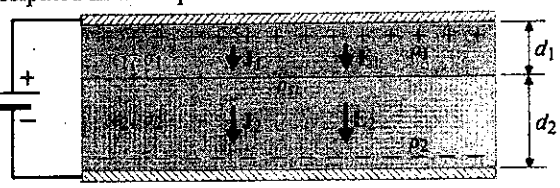
1. (10%) Write down the Maxwell's equations. State clearly the meaning of every physical symbol you write down.

2. (16%) A positive point charge Q is at the center of a spherical dielectric shell of an inner radius R_i and an outer radius R_o. The dielectric constant of the shell is ϵ. Calculate the electric field $\vec{E}$, the electric potential V, the electric flux density $\vec{D}$, and the polarization $\vec{P}$ as functions of the radial distance R. Also, draw your results with diagrams.

3. (20%) A point charge Q is at a distance d from the center of a grounded conducting sphere of radius a ($a < d$). Calculate
   (a) the charge distribution induced on the surface of the sphere, and
   (b) the total charge induced on the sphere by integrating the result of (a) over the surface of the sphere.

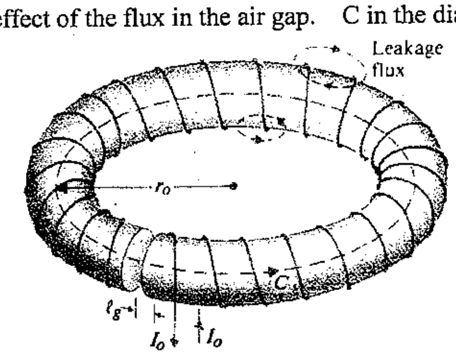
4. (20%) In a Hall-effect experiment, a current of 6.0 A sent length-wise through a conductor 1.0 cm wide, 4.0 cm long, and 10 μm thick produces a transverse (across the width) Hall voltage of 10 μV when a magnetic flux density of 1.5 T is passed perpendicularly through the thickness of the conductor. From these data, find
   (a) the drift velocity of the charge carriers and
   (b) the number density of charge carriers.
   (c) Show on a diagram the polarity of the Hall voltage with assumed current and magnetic flux directions, assuming also that the charge carriers are electrons.

5. (20%) An emf V is applied across a parallel-plate capacitor of area S. The space between the conducting plates is filled with two different lossy dielectrics of thicknesses d_1 and d_2, permittivities ϵ_1 and ϵ_2, and conductivities σ_1 and σ_2, respectively. Calculate
   (a) the current density between the plates, $\vec{J}_1$ and $\vec{J}_2$,
   (b) the electric field intensities in both dielectrics, $\vec{E}_1$ and $\vec{E}_2$,
   (c) the surface charge densities on the plates, ρ_1 and ρ_2, and at the interface, ρ_{si}, and
   (d) the power dissipated in the capacitor.



6. (14%) Assume that N turns of wire are wound around a toroidal core of a ferromagnetic material with permeability μ. The core has a mean radius r_0, a circular cross section of radius a ($a \ll r_0$), and a narrow air gap of length ℓ_g, as shown below. A steady current I_0 flows in the wire. Calculate
   (a) the magnetic flux density, $\vec{B}_f$, in the ferromagnetic core,
   (b) the magnetic field density, $\vec{H}_f$, in the core, and
   (c) the magnetic field density, $\vec{H}_g$, in the gap.

(Neglect the flux leakage and the fringing effect of the flux in the air gap. C in the diagram indicates a circular contour in the core.)



24% 1. Explain the following terms:

a. Bohr' Corresponding Principle.
b. Bohr' radius.
c. Bragg' law.
d. Davisson-Germer experiment.
e. the fine structure constant.
f. Fermi-Dirac probability function.
g. Larmor precession of an electron in oribital motion.
h. Thomas-Fermi approximation in multi-electron atoms.

16% 2. Using Einstein model and Debye model to calculate the molar heat capacity, Cv, at very low temperature and very high temperature.

note : $\int_0^{\infty} \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$

20% 3. Using Wilson-Sommerfeld Quantization rules to calculate
(1) the energy level of the motion of one dimensional harmonic oscillator.
(2) the energy level of a particle of mass m moves in one dimensional infinite square well of width a.

10% 4. For helium atom , write all possible singlet and trilet spin eigenfunctions.

15% 5. The eigenfunction of an electron in a hydrogen atom is described by

$$\varphi(\vec{r}) = \frac{1}{6}\left[4\varphi_{100}(\vec{r}) + 3\varphi_{211}(\vec{r}) - \varphi_{210}(\vec{r}) + \sqrt{10}\,\varphi_{21-1}(\vec{r})\right]$$

Find the expectation values of angular momentum L^2 and L_z.

15% 6. Calculate the Lande' g factor of the $^3P_1$ level in 2P3S configuration of ^6C atom, and evaluate the splitting of the level when the atom is in external magnetic field of 0.1 tesla.

note : $h = 6.626 \times 10^{-34}$ joul-sec

$m_e = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$ coulomb