國立中山大學九十四學年度碩士班招生考試試題

科目：近代物理【物理系碩士班】

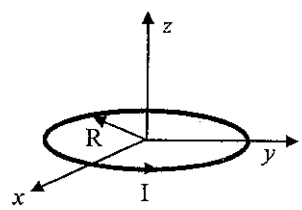
1. A photon of energy E strikes an electron at rest and undergoes pair production, producing a positive electron (positron) and an electron. The two electrons and the positron move off with identical momenta in the direction of the initial photon. (15%)
   (a) Find the kinetic energy of the three final particles.
   (b) Find the energy E of the photon.
2. If the energy density distribution function of the radiation in the cavity is defined as $u(\lambda)=\dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT}-1}$. (20%)
   (a) Find the long wavelength and the short wavelength limits of the energy density distribution function.
   (b) Derive the Stefan-Boltzmann law and express the Stefan's constant in terms of h, c, and k.
   [Hint: $\int_0^\infty \dfrac{x^3}{e^x-1}dx=\dfrac{\pi^4}{15}$].
3. (a) What is the Sommerfeld's quantization rule ?
   (b) Apply the quantization rule to the one-dimensional harmonic oscillator to find the energy. (15%)
4. A particle is bound in a one-dimensional potential well, its potential energy is given by $U(x)=0$ for $|x|\le L/2$, and $U(x)=\infty$ elsewhere. (20%)
   (a) Find the wave functions and energy for this particle.
   (b) Find the uncertainty in momentum, if the particle is in the ground state of this infinite well.
5. An electron in a hydrogen atom has the wavefunction $\Psi = N\,(3\Psi_{100} - 2\Psi_{211} + \Psi_{200} - 4\Psi_{320})$, where $\Psi_{n,l,m}$ are the solutions of Schrödinger's equation for the hydrogen atom. (20%)
   (a) Determine normalization constant N.
   (b) What is the expectation value of the energy?
   (c) What is the expectation value of L^2?
   (d) What is the expectation value of L_z?
   (e) What is the probability that a measurement will find the atom in the n = 2 state?
6. The density of silver is 10.5 g/cm^3 and its molecular weight is 107.9 g/mol. Compute the Fermi energy and the Fermi temperature for silver at 0 K. (10%)

Entrance Examination
(電磁學)

1. (25%) A sphere of radius R contains positive charges proportional to the radius, $\rho = Kr$ where K is a constant. Please find the electric field and potential inside and outside of the sphere. (Show all the details of calculation.)

2. (25%) A parallel plate capacitor is filled with a dielectric material (ε_r). When a battery of V is connected to the capacitor, please calculate the electric field in side the material, the bounded surface charge on the top (P point) of the dielectric material and the capacitance of the capacitor.

3. (25%) A ring with a radius R carries a constant current I flowing counterclockwise. Please find the magnetic vector $\vec{A}$, the magnetic flux density $\vec{B}$ and prove that $\vec{B} = \vec{\nabla} \times \vec{A}$ at the center of the ring.



4. (25%)

   (1) Please write down Maxwell equations and their names.

   (2) Please prove that the potential function in a static and a time varying cases are

   $\vec{E} = -\vec{\nabla} V$ and $\vec{E} = -\vec{\nabla} V - \dfrac{\partial \vec{A}}{\partial t}$ respectively.

   (3) What is the wave equations that can be derived from question (1)

   (4) What are boundary conditions at the interface where divided two dielectrics (ε_1, μ_1) and (ε_2, μ_2).

Show the details of your work.

1. (8%) In vacuum, Maxwell's equations become

$$\vec{\nabla}\bullet\vec{B}=0,\ \vec{\nabla}\bullet\vec{E}=0,\ \vec{\nabla}\times\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t},\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}.$$

Here $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic induction, ε_0 is the electric permittivity, and μ_0 is the magnetic permeability. Derive the electromagnetic wave equation

$$\vec{\nabla}\bullet\vec{\nabla}\vec{E}=\varepsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2}.$$

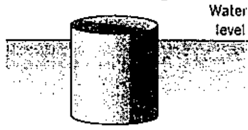
(Hint: $\vec{\nabla}\times(\vec{\nabla}\times\vec{V})=\vec{\nabla}\vec{\nabla}\bullet\vec{V}-\vec{\nabla}\bullet\vec{\nabla}\vec{V}$)

2. The force field acting on a two-dimensional linear oscillator may be described by $\vec{F}=-kx\hat{x}-ky\hat{y}$, k is a constant. Compare the work done moving against this force field when going from (1, 1) to (5, 5) by the following straight-line path: (2%)(a) (1, 1) → (5, 1) → (5, 5), (2%)(b) (1, 1) → (1, 5) → (5, 5), and (2%)(c) (1, 1) → (5, 5) along $x=y$. This means evaluating $-\int_{(1,1)}^{(5,5)}\vec{F}\bullet d\vec{r}$ along each path.

3. Given the pair of equations 2x − 3y = 4, and 6x − 9y = 12, (2%)(a) show that the determinant of the coefficients vanishes; (4%)(b) show that the numerator determinants also vanish; (4%)(c) find at least two solutions.

4. (10 points) Newton's law of cooling: A thermometer, reading 6 °C, is brought into a room whose temperature is 26 °C. One minute later, the thermometer reading is 10 °C. How long does it take until the reading is practically 26 °C, say, 25.9 °C?

5. (10 points) Archimedes's principle states that the buoyancy force equals the weight of the water displaced by the body (partly or totally submerged). Figure below shows a cylindrical buoy 70 cm in diameter standing in water with its axis vertical. When depressed slightly and released, its period of vibration is 1



sec. Find the weight of the buoy.

6. Given a general solution of the homogeneous system $y^{(h)}=c_1\begin{bmatrix}1\\1\end{bmatrix}e^{-2t}+c_2\begin{bmatrix}1\\-1\end{bmatrix}e^{-4t}$, find a general solution of

$$y' = Ay + g = \begin{bmatrix}-3 & 1\\ 1 & -3\end{bmatrix}\begin{bmatrix}y_1\\ y_2\end{bmatrix}+\begin{bmatrix}-6\\ 2\end{bmatrix}e^{-2t} \text{ using}$$

(8 points) (a) the method of variation of parameters

(8 points) (b) the method of diagonalization.

7. (10 points) Solve the initial value problem by the Laplace transform.

$$y'' + 16y = 4\delta(t-\pi),\ y(0) = 2,\ y'(0) = 0.$$

8. (10 points) Solve $(1-x^2)y'' - 2xy' + 2y = 0$ by the power series method.

9. (10 points) Find the potential inside and outside a spherical capacitor consisting of two metallic hemispheres of radius 1 ft separated by a small slit for reasons of insulation, if the upper hemisphere is kept at 150 volts and the lower is grounded.

10. (10 points) Use the residue theorem to evaluate the integral $\frac{z^2\sin z}{4z^2-1}, C:|z|=1$ over the given path C counterclockwise.