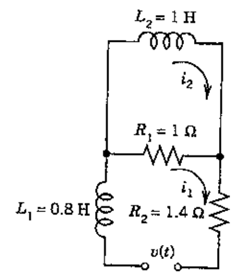
國立中山大學96學年度碩士班招生考試試題

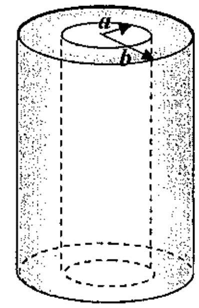
科目：應用數學【物理系碩士班】

Show the detail of your work.



Network

1. (20%) Find the currents $i_1(t)$ and $i_2(t)$ in the network shown in the right figure with L and R indicated on the figure. $v(t)$=100 volts in $0 \leqq t \leqq 0.5$ and 0 thereafter, and $i(0)$=0 and $i'(0)$=0. [You have to write the complete differential equations, the equations by applying Laplace transformation on these differential equations and the final solutions of this problem.]



2. (20%) An ***infinite*** long cylinder with inner and outer radii of a and b. When the radius is within a and b, a charge density ρ is uniform distributed. Please use Gauss Law [$\oint \vec{E} \bullet d\vec{a} = Q/\varepsilon_0$ where Q is the total charge contained inside the Gauss surface.] to find the electric field for $a < r < b$. [You have to write down the explicitly format of all parts in Gauss Law, explain clearly how to perform vector integration and calculate the final result.]

3. (20%) Please calculate the Fourier expansion of a triangle function by an even extension expansion.

$$f(x) = \begin{cases} \dfrac{2k}{L}x & \text{for } 0 < x < \dfrac{L}{2} \\ \dfrac{2k}{L}(1-x) & \text{for } -\dfrac{L}{2} < x < 0 \end{cases}$$

4. (30%) An electron is sitting on a two dimensional potential well where the potential inside the well (0<x<a and 0<y<a) is zero and ∞ elsewhere. Please find the lowest five bound energies, $E_{m,n}$, and the corresponding bond states, $\varphi_{m,n}$(x,y),of the electron.

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\varphi(x,y) + V(x,y)\varphi(x,y) = E\varphi(x,y)$$

$$V(x,y) = \begin{cases} 0 & \text{when } 0 < x < a \text{ and } 0 < y < a \\ \infty & \text{when elsewhere} \end{cases} \quad \text{and}$$

$$\varphi(0,y) = \varphi(a,y) = \varphi(x,0) = \varphi(x,a) = 0 \quad \text{and}$$

$$\varphi'(0,y) = \varphi'(a,y) = \varphi'(x,0) = \varphi'(x,a) = 0$$

5. (10%) Please calculate the Cauchy's integral $\oint_c g(z)dz$ where

$$g(z) = \frac{z^2+1}{z^2-1}.$$

1. In Fig. 1, $R_1 = 100\ \Omega$, $R_2 = R_3 = 50.0\ \Omega$, $R_4 = 75.0\Omega$, and the ideal battery has emf $\varepsilon = 6.0$ V. (a) What is the equivalent resistance? **[4 points]** What is the current i in (b) resistance R_1 **[4 points]**, (c) resistance R_2 **[4 points]**, (d) resistance R_3 **[4 points]**, and (e) resistance R_4? **[4 points]**

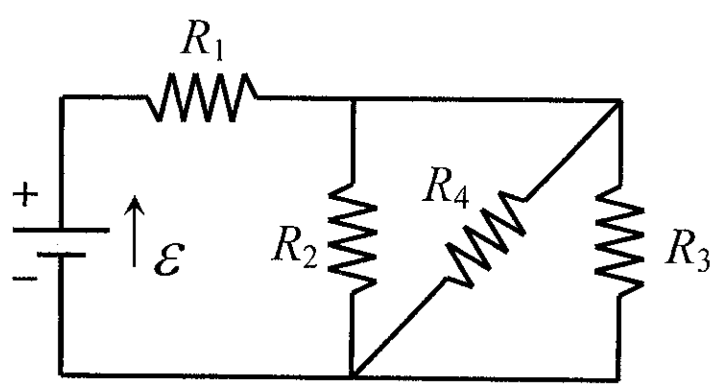


Fig. 1 (Problem 1)

2. A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as in Fig. 2). The shell carries no net charge. (a) Find the surface charge density σ at R, at a, and at b. **[6 points]** (b) Find the potential at the center, using infinity as the reference point. **[4 points]** (c) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as at infinity). How do your answers to (a) and (b) change? **[10 points]**

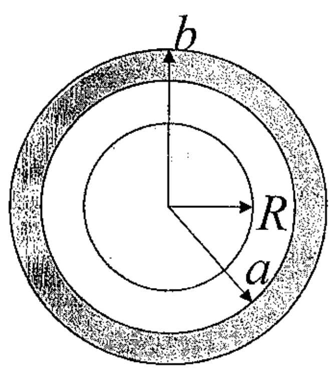


Fig. 2 (Problem 2)

3. In 1897 J.J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of "cathode rays" (actually, streams of electrons, with charge q and mass m) as follows: (a) First he passed the beam through uniform crossed electric and magnetic fields **E** and **B** (mutually perpendicular, and both of them perpendicular to the beam), and adjusted the electric field until he got zero deflection. What, then, was the speed of the particles (in terms of E and B) **[5 points]**? (b) Then he turned off the electric field, and measured the radius of curvature, R, of the beam, as deflected by the magnetic field alone. In terms of E, B, and R, what is the charge-to-mass ratio (q/m) of the particles **[5 points]**?

4. Figure 3 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume A = 115 cm^2, d = 1.24 cm, V_0 = 85.5 V, b = 0.78 cm, and κ = 2.61. (ε_0= 8.85 x 10^{-12} F/m) (a) What is the capacitance C_0 before the dielectric slab is inserted? **[4 points]** (b) What free charge appears on the plates? **[4 points]** (c) What is the electric field E_0 in the gaps between the plates and the dielectric slab? **[4 points]** (d) What is the electric field E_1 in the dielectric slab? **[4 points]** (e) What is the potential difference V between the plates after the slab has been introduced? **[2 points]** (f) What is the capacitance with the slab in place between the plates of the capacitor? **[2 points]**

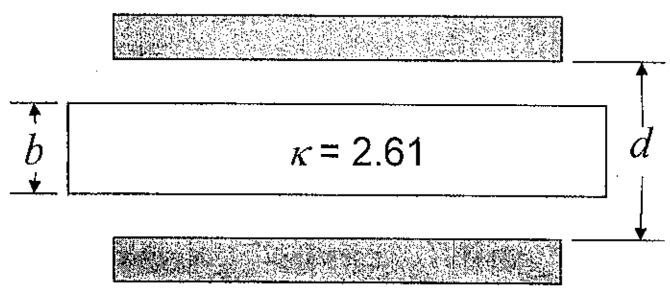


Fig. 3 (Problem 4)

5. The magnetic field on the axis of a circular current loop is far from uniform. You can produce a more nearly uniform field by using two such loops a distance d apart (**Fig. 4, Helmholtz coil**). (a) Find the field (B) as a function of z, and show that $\partial B/\partial z$ is zero at the point midway between them (z = 0). **[5 points]** (b) Determine d such that $\partial^2 B/\partial z^2 = 0$ at the midpoint, and find the resulting magnetic field at the center. **[5 points]**

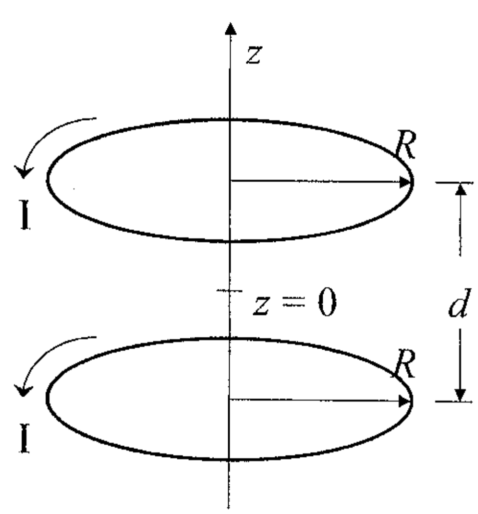


Fig. 4 (Problem 5)

6. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (Fig. 5). A resistor $\boldsymbol{R}$ is connected across the rails and a uniform magnetic field $\mathbf{B}$, pointing into the page, fills the entire region. (a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow **[5 points]**? (b) What is the magnetic force on the bar? In what direction **[5 points]**? (c) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t **[5 points]**? (d) The initial kinetic energy of the bar was $(mv_0^2/2)$. Check that the energy delivered to the resistor is exactly $(mv_0^2/2)$. **[5 points]**

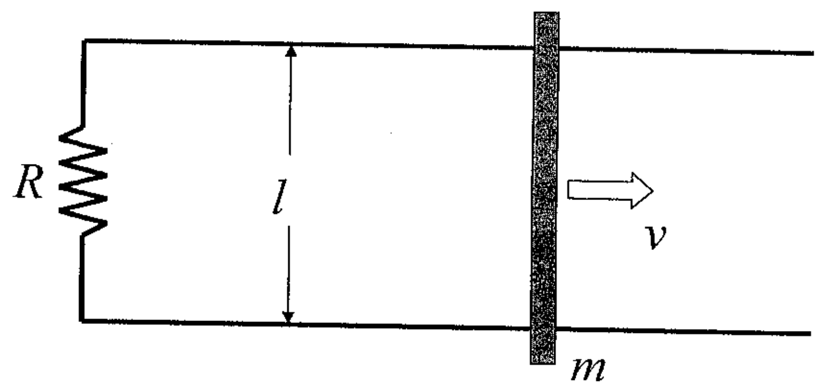


Fig. 5 (Problem 6)

國立中山大學96學年度碩士班招生考試試題

科目：近代物理【物理系碩士班】

1. Write down the following four-vectors(vector in the four dimensional space-time) which are transformed under the Lorentz transformation between two frames with constant relative velocity $\vec{V}=v\vec{i}$, (a)four-velocity, (b)four-momentum;(rest mass m_0), and (c)four-current-density. (d)Show $E^2 = c^2p^2 + m_0^2c^4$. (no need to working (a), (b), and (c) out!)(5%,3%,5%,5%)

2. Write down the following quantities defined in quantum mechanics, (a)Bohr radius, (b)elctron's Compton wavelength, (c)the uncertainty of position, and (d) the probability-current-density. (no need to working out)(3%,3%,3%,5%)

3. (a)What is the Fermi energy? (b)What is the Bloch wave function? (c)What is the Bose-Einstein Condensation?(4%,4%,4%)

4. (a)What is the β decay? (b)Why do most stable nuclei have equal number of protons and neutrons?(5%,5%)

5. A particle is trapped in a one-dimensional region of size L. In its second excited state($n = 3$). (a)Find the probability of finding particle in a region from $\frac{L}{4}$ to $\frac{L}{3}$. (b)Find the expectation value of momentum. (c)Find the particle's energy.(mass of particle, m)(7%,7%,7%)

6. Consider the normal Zeeman effect applied to the $3d$ to $2p$ transition. (a)Sketch an energy-level diagram and show all possible transitions. (b)How many different transiton energies emitted.(7%,3%)

7. Why is there a spin-orbital interaction in an atom? What does its hamiltonian look like for a hydrogen atom?(5%,5%)

8. What are the n-type and p-type semiconductors?(5%)