1. Find the limit \( \lim_{x \to 0} \frac{1 - \cos x}{\sin x} \).

2. Find the second derivative of \( f(x) = 20\sqrt{x} \).

3. True or False? Every \( n \)-th degree polynomial has \( (n - 1) \) critical numbers.

4. Find the limit \( \lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - x}} \).

5. Find the indefinite integral \( \int (3 - x^2)^3 \, dx \).

6. Find \( F(x) = \int_0^x t^2 \sqrt{1 + t^3} \, dt \).

7. Find the limit \( \lim_{x \to \infty} \sinh x \).

8. Find the derivative of \( f(x) = 2 \arcsin(x - 1) \).

9. Find the arc length of the graph of the function \( y = \frac{3}{2} x^{2/3} \) on \([1, 8]\).

10. Find the integral \( \int \frac{\sin x}{\cos x (\cos x - 1)} \, dx \).

11. Find the integral \( \int x^2 e^{2x} \, dx \).

12. Determine the convergence or divergence of the series \( \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \).

13. Find the Maclaurin series for the function \( f(x) = \sqrt{1 + x} \).

14. Find an equation of the parabola with vertex \((5, 4)\) and focus \((3, 4)\).

15. Find the area of the surface formed by revolving the curve \( r = 6 \cos \theta, \, 0 \leq \theta \leq \frac{\pi}{2} \) about the polar axis.

16. Find the angle \( \theta \) between the vectors \( \mathbf{u} = 5[\cos(3\pi/4)i + \sin(3\pi/4)j] \) and \( \mathbf{v} = 2[\cos(2\pi/3)i + \sin(2\pi/3)j] \).

17. Evaluate \( \int (\cos ti + t \cos t j) \, dt \).

18. Find the directional derivative of \( g(x, y) = x^2 + y^2 + 1 \) at \( P(1, 2) \) in the direction of \( Q(2, 3) \).

19. True or False? If \( f \) has a relative maximum at \((x_0, y_0, z_0)\), then \( f_x(x_0, y_0) = f_y(x_0, y_0) = 0 \).

20. Find the area of region inside the circle \( r = 2 \cos \theta \) and outside the circle \( r = 1 \).
一、選單題(每題 5 分，不倒扣):

1. A particle with a mass of 2 kg is acted on by \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) to accelerate at \( 4\mathbf{i} - 7\mathbf{j} + 3\mathbf{k} \) (m/s^2). It is known that
   \( \mathbf{F}_1 = 2\mathbf{i} + 5\mathbf{j} + 9\mathbf{k} \) (N), then \( \mathbf{F}_2 = \) (A) \( 2\mathbf{i} - 7\mathbf{j} - 3\mathbf{k} \), (B) \( 2\mathbf{i} - 5\mathbf{j} + 5\mathbf{k} \), (C) \( \mathbf{F}_2 = 6\mathbf{i} - 5\mathbf{j} + 5\mathbf{k} \), (D) \( \mathbf{F}_2 = 6\mathbf{i} - 7\mathbf{j} - 3\mathbf{k} \), (E) \( \mathbf{F}_2 = 2\mathbf{i} - 5\mathbf{j} \) (N).

2. A particle moves along the x-axis according to the equation \( 7t = 3x^2 + 8x + 5 \), where \( x \) is the position in meter and \( t \) is the time in second, find its speed at \( x = 1.0 \) m.
   (A) 2.0, (B) 1.5, (C) 1, (D) 0.5, (E) 0.1 m/s^2.

3. Which in the following does not represent the dimension of force?
   (A) \( ma \), (B) \( kv \), (C) \( qE \), (D) \( qB \), (E) \( mv^2/x \), \( m \): mass, \( x \): position, \( v \): speed, \( a \): acceleration, \( k \): force constant, \( q \): charge, \( E \): electric field, \( B \): magnetic induction.

4. A solid wheel with mass \( M \), radius \( R \), rolls without sliding on a horizontal surface and its rotational inertia is \( MR^2/2 \). If the center of mass is accelerating at \( a \). Find the applied force \( F \) acting on the axle and the frictional force \( f \) on the wheel surface. \( F \), \( f \) = (A)\( (Ma, 0) \), (B)\( (Ma, Ma/2) \), (C)\( (2Ma, Ma) \), (D)\( (2Ma, Ma/2) \), (E)\( (3Ma/2, Ma/2) \).

5. A wheel takes 3.00 s to rotate 36.0 rounds. Its angular speed at the end of the 3 s interval is 27 \( \pi \) rad/s. What is the constant angular acceleration of the wheel?
   (A) 2.0, (B) 1.0, (C) 3.0, (D) 0.5, (E) 4.0 (rad/s^2).

6. A charge of 10 C is first put on a spherical conducting shell and then another point charge of \( -3 \) C is put at the center. Find the net charge on the outer surface of the shell:
   (A) \(-7 \) C, (B) \(-3 \) C, (C) 0 C, (D) \(+3 \) C, (E) \(+7 \) C.

7. A capacitor of capacitance \( C \) and an inductor of inductance \( L \) are connected on both ends, the resonance angular frequency is:
   (A) \( LC \), (B)\( (LC)^{1/2} \), (C) \( LC^2 \), (D) \( 1/LC^{1/2} \), (E) \( L/C \).

8. A magnetic field \( B \) passing in perpendicular through a square wire loop of area \( A \). The magnetic flux through the loop is:
   (A) 0, (B) \( BA/2 \), (C) \( BA \), (D) \( 2BA \), (E) \( 3BA \).

9. In the right figure, \( R_1 = 5 \) \( \Omega \), \( R_2 = 10 \) \( \Omega \), \( R_3 = 15 \) \( \Omega \), \( C_1 = 5 \) \( \mu F \), \( C_2 = 10 \) \( \mu F \) and the ideal battery has an emf \( \Delta = 20 \) V. In steady state, the total energy stored in the two capacitors is:
   (A) \( 2.78 \times 10^{-5} \), (B) \( 1.12 \times 10^{-4} \), (C) \( 2.22 \times 10^{-4} \), (D) \( 2.50 \times 10^{-4} \), (E) \( 4.72 \times 10^{-4} \) (J).

10. A monochromatic light (\( \lambda = 560 \) nm) is incident on a thin film with refractive index \( n = 1.40 \). How thick must the film be in order for destructive interference to occur when reflected?
    (A) 100, (B) 200, (C) 150, (D) 250, (E) 50 (nm).

11. Assume the pupil diameter is 0.50 cm and \( n = 1.22 \) for the naked eye. What is the maximal distance if one is to distinguish between two blue lights (\( \lambda = 500 \) nm) separated by 1.5 m?
    (A) 6.0, (B) 12, (C) 9.0, (D) 3.0, (E) 15 (km).

12. For diffraction of waves by a single slit of width \( a \) at a distance \( D \) from the screen, which of the following is correct?
    (A) the first maximum occurs at \( a \sin \theta = \lambda \), (B) smaller \( a \) will result in larger separations between minima, (C) the intensity of each maximum is the same, (D) the diffraction is more easily observed for lights of shorter wavelength than those of longer wavelength, (E) the first maximum occurs at \( a \sin \theta = 3\lambda/2 \).
13. Accelerated by a potential difference \( V \), two charged particles of the same charge \( q \) but different masses \( m_1 \) and \( m_2 \) enter a uniform field \( B' \) following a circular path illustrate in the right figure. What is the following are correct:
(A) The kinetic energy is \( qV' B' \) for both particles, (B) the speed of \( m_1 \) is \( (2qV/m_1)^{1/2} \), (C) the radius of the path in \( B' \) for \( m_2 \) is \( (m_2 V/qB'^2)^{1/2} \), (D) \( r_1/r_2 = (m_2/m_1)^{1/2} \), (E) \( r_1/r_2 = (m_1/m_2)^{1/2} \).

14. A solid sphere of radius \( R \) has a volume charge density \( \rho = \alpha r/R \), where \( r \) is the radial distance from the sphere’s center. What’s correct?
(A) The Gauss’s Theorem is not applicable, (B) the total charges on the sphere is \( \alpha R^3 \), (C) the magnitude of the electric field at \( r = 0 \) is zero, (D) the magnitude of the electric field at \( r = R \) is \( \frac{1}{16} \frac{1}{\varepsilon_0} \), (E) the magnitude of the electric field at \( r = R \) is \( \frac{\alpha R}{\varepsilon_0} \).

15. Which of the following are correct?
(A) There can be no electric field inside a solid conducting sphere, (B) the Gauss’s Theorem of magnetism suggests that no magnetic monopole exists, (C) electric current flowing in a wire will produce a magnetic field, (D) a static magnetic field threaded through a loop will cause an emf, (E) alternating changing electric and magnetic fields will generate an electromagnetic wave.

16. A beam of unpolarized light is sent into a sequence of three polarizing sheets. The polarization angles are fixed at \( \theta_1 \) and \( \theta_3 \) for sheet 1 and 3, respectively, but for sheet 2 the angle \( \theta_2 \) is variable. The transmitted intensity of the light as a function of \( \theta_2 \) is shown to the right. (A)
Select the correct answers from below: (A) \( \theta_1 = 90^\circ \) or \( \theta_3 = 90^\circ \), (B) transmission rate of the unpolarized light after sheet 1 is 1/2, (C) \( |\theta_2 - \theta_3| = 90^\circ \), (D) the transmitted intensity is 0 if one takes out the sheet 2, (E) transmission rate is 1/8 if \( \theta_2 = 30^\circ \).

17. A setup to test photoelectric effect is illustrated to the right.
(A) The kinetic energy \( K_{\text{max}} \) of the photoelectron is proportional to the stopping potential, (B) for a light of given frequency, \( K_{\text{max}} \) does not depend on the intensity of the light source, (C) increasing the intensity of the light source would cause larger \( K_{\text{max}} \), (D) the slope of the "stopping potential vs. incident light frequency" plot depends on the metals, (E) the work function of the target metal can be derived (\( I\) is the target, \( C \) is the cathode in the diagram).

三、計算題(10 分):

An oscillator antenna, as shown in the right figure, is connected through a transmission line and a transformer to an \( LCR \) oscillator circuit. If \( R \) is so small that one can take \( R=0 \), what is the (a) speed (4%), (b) frequency (3%), and (c) wavelength of the electromagnetic wave emitted by the antenna assuming that \( L = 0.20 \mu \text{H} \) and \( C = 45.0 \text{ pF} \) (3%)?