解下列所有問題，並詳細寫計算過程與說明。不需依題號順序作答，但請標示所給之解所對應之題號。下列問題中將使用符號:

\( \mathbb{R} \) = the set of all real numbers; \( e \) = the base of the natural exponential function.

1. Let \( f(x, y, z) \) be a differentiable real valued function of three real variables with the first partial derivatives \( f_x(-4, 3, 2) = -1, f_y(-4, 3, 2) = 3 \) and \( f_z(-4, 3, 2) = -2 \). Let

\[ g(u, v) = f(uv, \sqrt{1 + u^2 + v^2}, 3u + 2v) \quad \text{for} \quad u, v \in \mathbb{R}. \]

Evaluate the first partial derivative \( \frac{\partial g}{\partial u}(2, -2). \) (6 分)

2. Let \( f(x, y) \) be a differentiable real valued function of two real variables, and let \( \alpha(t) \) and \( \beta(t) \) be differentiable curves lying on the graph of \( f \) with \( \alpha(0) = \beta(0) = (a, b, f(a, b)) \), where \( a, b \in \mathbb{R} \). Assume that \( \alpha'(0) = (4, 5, 3) \) and \( \beta'(0) = (2, 3, 2) \). Evaluate the first partial derivatives \( f_x(a, b) \) and \( f_y(a, b) \).

(6 分)

3. Let \( f(x) = (x^2 - 2x + 4)^{-1} \) for \( x \in \mathbb{R} \). Evaluate the 6-th order derivative \( f^{(6)}(1) \) of \( f \) at 1.

(8 分)

4. Let \( f(x) = \int_0^{-\frac{x}{1+3x^2}} (\frac{1}{e^{x^2}} - 1) \, dt \) for \( x \in \mathbb{R} \). Evaluate \( \lim_{x \to 0} \frac{f(x)}{x^2 + 2 \cos x - 2} \)

(8 分)

5. Evaluate the following integrals:

(a) \( \int_0^1 \frac{1 + x}{1 + \sqrt[3]{x}} \, dx \) (6 分)

(b) \( \int_0^{\sqrt{\frac{1}{2}}} \frac{\sqrt{1 - x}}{\sqrt{1 + x}} \, dx \) (8 分)

6. Evaluate the following iterated integrals:

(a) \( \int_0^1 \int_0^\pi x^2 \sin(xy) \, dx \, dy \) (8 分)

(b) \( \int_0^1 \int_x^1 \frac{y}{1 + y^6} \, dy \, dx \) (8 分)

(c) \( \int_0^{\frac{1}{2}} \int_y^{\sqrt{1-y^2}} \frac{xy}{1 + x^2 + y^2} \, dx \, dy \) (8 分)

7. Evaluate the triple integral \( \iiint_{\Omega} x^2 \, dV \), where

\[ \Omega = \{(x, y, z) : 0 \leq z \leq \min(\sqrt[3]{3(x^2 + y^2)}, \sqrt{4 - x^2 - y^2}) \text{ and } 0 \leq y \leq x \} \]

(10 分)

8. Let \( f(x, y) = 2x^2 + y^2 \) for \( x, y \in \mathbb{R} \). Find the absolute extrema of \( f \) subject to the constraint \( 2x^2 - \sqrt{2}xy + y^2 = 1 \).

(12 分)

9. For every integer \( n > 0 \), let \( a_n = \frac{\ln(n+8)}{\sqrt{n+8}} \). Find the interval of convergence of the power series \( \sum_{n=1}^{\infty} a_n(3-x)^n \).

(12 分)
1. (10%) Two forces of magnitude 50 N, as shown in the figure below, act on a cylinder of radius 4 m and mass 6.25 kg. The cylinder, which is initially at rest, sits on a frictionless surface. After 1 second, the velocity and angular velocity of the cylinder in m/s and rad/s are respectively. (a) \( v = 0; \omega = 0 \); (b) \( v = 0; \omega = 4 \); (c) \( v = 0; \omega = 8 \); (d) \( v = 8; \omega = 8 \); (e) \( v = 16; \omega = 8 \).

2. (10%) A 2.5-kg object suspended from the ceiling by a string that has a length of 2.5 m is released from rest with the string 40° below the horizontal position. What is the tension in the string at the instant when the object passes through its lowest position? (a) 11N; (b) 25N; (c) 42N; (d) 18N; (e) 32N

3. (10%) A wheel rotates about a fixed axis with a constant angular acceleration of 4.0 rad/s². The diameter of the wheel is 40 cm. What is the linear speed of a point on the rim of this wheel at an instant when that point has a total linear acceleration with a magnitude of 1.2 m/s²? (a) 39 cm/s; (b) 42 cm/s; (c) 45 cm/s; (d) 35 cm/s; (e) 53 cm/s

4. (10%) A solid sphere, spherical shell, solid cylinder and a cylindrical shell all have the same mass \( m \) and radius \( R \). If they are all released from rest at the same elevation and roll without slipping, which reaches the bottom of an inclined plane first? (a) solid sphere; (b) spherical shell; (c) solid cylinder; (d) cylindrical shell; (e) all take the same time

5. (10%) An ideal gas is allowed to undergo a free expansion. If its initial volume is \( V_1 \) and its final volume is \( V_2 \), the change in entropy is (a) \( nR \ln(V_2 / V_1) \); (b) \( nRT \ln(V_2 / V_1) \); (c) \( nk \ln(V_2 / V_1) \); (d) 0; (e) \( nR V_2 / V_1 \).

6. (10%) The open switch in Figure 2 is thrown closed at \( t = 0 \). Before the switch is closed, the capacitor is uncharged and all currents are zero. Determine the currents in \( L, C, \) and \( R \), the emf across \( L \), and the potential differences across \( C \) and \( R \) (a) at the instant after the switch is closed (5%) and (b) long after it is closed (5%).

7. (10%) Consider the hemispherical closed surface in Figure 3. The hemisphere is in a uniform magnetic field that makes an angle \( \theta \) with the vertical. Calculate the magnetic flux through the hemispherical surface \( S_2 \).
8. (15%) Two blocks of mass $m_1$ and $m_2$ are connected by a massless string over a pulley in the shape of a solid disk having radius $R$ and mass $M$. The fixed, wedge-shaped ramp makes an angle of $\theta$ as shown in Fig. 4. The coefficient of kinetic friction is $\mu_k$ for both blocks. (a) Draw force diagrams of both blocks and of the pulley. (5%) Determine (b) acceleration of the two blocks (5%) and (c) the tensions in the string on both sides of the pulley. (5%)

9. (15%) A conducting bar of length $l$ rotates with a constant angular speed $\omega$ about a pivot at one end. A uniform magnetic field $\mathbf{B}$ is directed perpendicular to the plane of rotation as in Fig. 5. Find the emf induced between the ends of the bar. Point out which one is high potential end, $O$ or $P$ sides.