解下列所有問題，並詳細寫計算過程與說明。不需依題號順序作答，但請標示所給之解所對應之題號。下列問題中將使用符號:

\[ \mathbb{R} = \text{the set of all real numbers}; \quad e = \text{the base of the natural exponential function}. \]

1. Let \( p, q, r \in \mathbb{R} \) be constants, and let \( f(x) = \frac{1}{4}x^4 + px^3 + qx^2 + 4x + r \) for \( x \in \mathbb{R} \). Assume that \( f(1) \) and \( f(2) \) are relative extrema of \( f \), and that

\[
\min \{ f(x) : -2 \leq x \leq 3 \} = \frac{-8}{3}.
\]

Let \( M = \max \{ f(x) : -2 \leq x \leq 3 \} \). Evaluate the values of \( p, q, r, \) and \( M \). (8 分)

2. Let \( f(x) = 15x^{7/3} - 21x^{5/3} + 6 \) for \( x \in \mathbb{R} \), and let \( C \) denote the graph of \( f \). Let \( m \) be the number of points of inflection of \( C \), and let \( n \) be the number of points where \( C \) intersects the \( x \)-axis. Evaluate the values of \( m \) and \( n \). (10 分)

3. Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be the function defined by

\[
f(x) = \int_0^x \frac{2 - t}{t^2 - 2t + 4} \, dt \quad \text{for } x \in \mathbb{R}.
\]

For every integer \( n > 0 \), let \( f^{(n)}(1) \) denote the \( n \)-th derivative of \( f \) at 1. Evaluate the values of the derivatives \( f^{(8)}(1) \) and \( f^{(8)}(1) \). (8 分)

4. Let \( C \) be the curve of equation \( y = \sqrt{x^2 + \frac{1}{\sqrt{e^x}}} \) for \( 0 \leq x \leq \ln 3 \). Evaluate the length of \( C \). (6 分)

5. Evaluate the following integrals:

(a) \( \int_4^{12} \frac{1}{(x+4)\sqrt{x}} \, dx \) (6 分)

(b) \( \int_0^4 \frac{1}{(x+1)\sqrt{1-x^2}} \, dx \) (8 分)

6. Let \( f(x, y) = \frac{x^2 y}{1 + x^2 + y^2} \) for \( x, y \in \mathbb{R} \), and \( g(u, v) = f(uv, u^2 + 2v^2) \) for \( u, v \in \mathbb{R} \).

Evaluate the first partial derivative \( \frac{\partial g}{\partial v}(-1, -1) \). (6 分)

7. Evaluate the following iterated integrals:

(a) \( \int_0^{\frac{\sqrt{2}}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{x} \cos(\sqrt{x}y) \, dx \, dy \) (6 分)

(b) \( \int_0^{\frac{\sqrt{3}}{2}} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{4y}{1 + \sqrt{x^2 + y^2}} \, dx \, dy \) (6 分)
8. Let $\Omega$ be the solid tetrahedron with vertices at the points $(1, 0, 0), (0, 1, 0), (1, 1, 0)$ and $(1, 1, 1)$. Evaluate the triple integral $\iiint_{\Omega} xy \, dV$. 

9. Evaluate the triple integral $\iiint_{\Omega} \frac{1}{1 + x^2 + y^2 + z^2} \, dV$, where

$$\Omega = \{(x, y, z) : x, y, z \in \mathbb{R}, \frac{\sqrt{x^2 + y^2}}{\sqrt{3}} \leq z \leq \sqrt{1 - x^2 - y^2} \text{ and } 0 \leq y \leq x\}.$$ 

10. For every integer $n > 0$, let $a_n = \frac{1}{\sqrt{n}} - \sin \frac{1}{\sqrt{n}}$. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} a_n(x + 2)^n$. 

11. Use the definition to evaluate the improper integral $\int_{1}^{\infty} \frac{\ln(1 + x)}{x^3} \, dx$. 

背面有題
1. In Faraday’s experiment, he attached a primary coil to a switch and battery and a secondary coil to a sensitive ammeter as shown here. Both coils are wrapped around an iron ring. When the switch is closed, the ammeter in the secondary circuit deflects momentarily. What is the deflected direction of the Ammeter’s pointer (A or B) and why? (10%)

2. An unpolarized beam of light is incident on a stack of ideal polarizing filters. The axis of the first filter is perpendicular to the axis of the last filter in the stack. Three filters are in the stack, each with its transmission axis at 45.0° relative to the preceding filter. Find the fraction by which the transmitted beam’s intensity is reduced. (10%)

3. Two balls with masses $M$ and $m$ are connected by a rigid rod of length $L$ and negligible mass as in Fig.2. For an axis perpendicular to the rod, calculate the moment of inertia of the system which has the minimum value. (10%)

4. Two slits are illuminated with green light ($\lambda = 540$ nm). The slits are 0.05 mm apart and the distance to the screen is 1.5 m. At what distance (in mm) from the central maximum on the screen is the average intensity 50% of the intensity of the central maximum? (10%)

5. The light intensity incident on a metallic surface with a work function of 3 eV produces photoelectrons with a maximum kinetic energy of 2 eV. The frequency of the light is doubled. Calculate the maximum kinetic energy (in eV). (10%)

6. Consider the hemispherical closed surface in Fig. 3. The hemisphere is in a uniform magnetic field that makes an angle $\theta$ with the vertical. Calculate the magnetic flux through the hemispherical surface $S_2$. (10%)

7. For a adiabatic free expansion of an ideal gas, suppose the ideal gas expands to four times its initial volume. For this process, the initial and final temperatures are the same. Calculate the entropy change for the gas. (10%)

8. A particle of mass $m$ is attached between two identical springs on a horizontal frictionless tabletop. The springs have force constant $k$ and each is initially unstressed. (a) If the particle is pulled a distance $x$ along a direction perpendicular to the initial configuration of the springs, as in Fig.4, show that the potential energy of the system is $U(x) = kx^2 + 2kL(L - \sqrt{x^2 + L^2})$. (8%) (b) Make a plot of $U(x)$ versus $x$ and identify all equilibrium points. (7%)

9. A projectile of mass $m$ moves to the right with a speed $v_1$ (Fig. 5a). The projectile strikes and sticks to the end of a stationary rod of mass $M$, length $d$, pivoted about a frictionless axle through its center (Fig. 5b). (a) Find the angular speed of the system right after the collision. (8%) (b) Determine the fractional loss in mechanical energy due to the collision. (7%)