1. (15pts) A vector field is given as \( \vec{G}(x, y, z) = \vec{a}_x(x - yz) + \vec{a}_y(y^2 - xz) + \vec{a}_z(z^2 - xy) \).

Find
(a) line integral from \((0, 0, 0)\) to \((1, 1, 1)\) along the straight line
(b) line integral from \((0, 0, 0)\) to \((1, 1, 1)\) along the curve \( y = x^2, z = x \)
(c) Does there exist a scalar field \( \Phi \) such that \( \vec{G} = \nabla \Phi \)? If so, determine \( \Phi \). If not, give the reason.

2. (10pts) Let \( x(t) \leftrightarrow X(j\omega), \ y(t) \leftrightarrow Y(j\omega), \) and \( z(t) \leftrightarrow Z(j\omega) \) denote Fourier transform pairs, related by
\[
Z(j\omega) = \frac{1}{\omega} \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt, \quad Z(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z(j\omega)e^{j\omega t} d\omega
\]
If \( Z(j\omega) = X(j\omega)Y(j\omega) \), express \( z(t) \) in terms of \( x(t) \) and \( y(t) \).

3. (10pts) An LTI system has impulse response \( h(t) \). The output of the system \( y(t) \) due to input \( x(t) \) is then
\[
y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau.
\]
(a) If \( h(t) = \delta(t) \), the system is called an identity system. Explain why it is so named from the input-output relation.
(b) If \( h(t) = \delta'(t) \) (derivative), determine \( y(t) \). You can derive your answer mathematically or by reasoning.

4. (20%) There are three questions as follows.
(a) Let \( X \) and \( Y \) be two subspaces of \( \mathbb{R}^n \). Write the definition of \( X \) and \( Y \) being orthogonal.
(b) Let \( X \) and \( Y \) be two orthogonal subspaces of \( \mathbb{R}^n \). Show that \( X \cap Y = \{0\} \).
(c) Let \( A \in \mathbb{R}^{m \times n} \) with \( \text{rank}(A) = r \), and let \( \{x_1, \ldots, x_r\} \) be a basis for \( R(A^T) \), the range of \( A^T \). Is \( \{Ax_1, \ldots, Ax_r\} \) a basis for \( R(A) \)? If the answer is YES, give a proof for it. Otherwise, provide a counterexample for it.

5. (10%) Given \( z_0 \in \mathbb{C} \), let \( \alpha := |z_0|, \beta := \Re(z_0), \) and \( \gamma := \Im(z_0) \).

Then solutions to the equation \( z^2 = z_0 \) are
\[
(a) \pm \frac{1}{\sqrt{2}}(\sqrt{\beta + \gamma} + i\sqrt{\beta - \gamma}) \\
(b) \pm \frac{1}{\sqrt{2}}(\sqrt{\beta + i\gamma}) \\
(c) \pm \frac{1}{\sqrt{2}}(\sqrt{\beta - \gamma} + i\sqrt{-\beta - \gamma}) \\
(d) \pm \frac{1}{\sqrt{2}}(\sqrt{\alpha + \beta} + i\sqrt{-\alpha - \beta}) \\
(e) \pm \frac{1}{\sqrt{2}}(\sqrt{\alpha + \gamma} + i\sqrt{\alpha - \gamma}) \\
(f) \pm \frac{1}{\sqrt{2}}(\sqrt{\alpha + i\gamma}) \\
(g) \pm \frac{1}{\sqrt{2}}(\sqrt{\alpha - \gamma} + i\sqrt{-\alpha - \beta}) \\
(h) \pm \frac{1}{\sqrt{2}}(\sqrt{-\alpha + \beta} + i\sqrt{-\alpha - \beta}).
\]

Please choose your answer(s) and give all the necessary details.

6. (15%) Find the general solution of the following differential equation:
\[
y' = (x + y + 1)^2
\]

7. (20%) Use the Laplace transform to solve the following initial value problem:
\[
y'' + 4y = \sin t, \quad u(t - 2\pi), \quad y(0) = 1, \quad y'(0) = 0.
\]
一、For the circuit shown in Fig. 1, (a) What is the gain \( v_o / v_i \)? (b) For op amps using \( \pm 15 \)-V supplies that limit at \( \pm 13 \) V, what is the largest sine wave you can provide across \( R_o \)? (c) Using 1 kΩ as the smallest resistor values that make \( v_o / v_i = 10 \) V. (8%\%, 4\%, 8\%)

二、(a) Plot and explain the difference of the voltage transfer characteristics of an NMOS inverter with a resistive load, an NMOS inverter with a depletion load, and a CMOS inverter together (i.e. in the same \( v_o - v_i \) Figure). (b) What is Noise Margins? Take CMOS inverter voltage transfer characteristics as an illustrative and define the noise margins on it. (8%\%, 8\%)

三、Introducing a new dominant pole to an unstable three-pole feedback amplifier, we can obtain a stable feedback amplifier. (a) What is the Nyquist stability criteria, gain margin and phase margin? (b) Plot the Bode plots of loop gain magnitude and phase for the three-pole amplifier, before frequency compensation, and after frequency compensation. (c) Same as in (b) plot the Nyquist plots for the amplifier before and after the frequency compensation. (4%\%, 8\%, 8\%)

四、In Fig. 4 a BiCMOS double cascode constant-current source is shown. (a) Plot the equivalent circuit for determining output impedance of the BiCMOS double cascode current source and (b) Derive the output resistance. (c) Why the constant current source need extremely large output resistance? (4%\%, 8\%, 4\%)

五、The op-amp in the diff-amp shown in Fig. 5, is ideal. (a) Derive the expressions of common-mode gain, \( A_{cm} \) and differential-mode gain, \( A_d \). (b) If the tolerance of each resistor is \( \pm 2\% \), determine the minimum value of CMRR. (8%\%, 8\%)

六、Consider the circuit shown in Fig. 6. Signals \( \phi_1 \) and \( \phi_2 \) are nonoverlapping clock signals. (a) Describe the operation of the circuit and the logic function implemented. (b) Discuss any possible relationship between the width-to-length ratios of the load and driver transistors for “proper” circuit operation. (8%\%, 4\%)
1. Derive the expression of the Hall voltage $V_H$ induced on both sides of a P-type semiconductor with a given current $I_T$ and the magnetic field $B$. The width of the semiconductor is $W$. The thickness is $d$, and the length is $L$. (17%)

2. Consider a p-n heterojunction with dielectric constants $\varepsilon_P$ and $\varepsilon_n$, and doping concentrations $N_d$ and $N_a$. Find the expression of the depletion width $W$ at the junction with respect to an applied voltage $V$. (17%)

3. The Schottky barrier height, $\phi_{BS}$, of a metal/n-GaAs MESFET is 0.9 volts. The channel doping is $N_d = 1.5 \times 10^{16} \text{cm}^{-2}$ and the channel thickness is $d = 0.5 \mu\text{m}$. The temperature $T$ is 300K. Calculate the pinch off voltage $V_p$ and determine the FET is enhancement type or depletion type. ($\varepsilon = 13.1 \varepsilon_0, \varepsilon = 8.85 \times 10^{-14} \text{F/cm}^2 N_c = 4.7 \times 10^{17} \text{cm}^{-3}$) (17%)

4. A MOS transistor with a p-type-polysilicon gate is fabricated on a p-type silicon substrate with $N_a = 2 \times 10^{19} \text{cm}^{-3}$. The oxide thickness is $t_{ox} = 650 \text{Å}$ and the equivalent fixed oxide charge is $Q_{ox} = 2 \times 10^{11} \text{cm}^{-2}$. (Si : $n_i = 1.5 \times 10^{10} \text{cm}^{-3}$, $\varepsilon_{Si} = 11.8$, $\varepsilon_{SiO_2} = 3.9$, $kT/q = 0.0259 \text{V}, E_g = 1.12 \text{eV}$) (17%)

   (a) Calculate the threshold voltage.
   (b) Calculate the threshold voltage difference $\Delta V_T$ when the bias voltage $V_{SS}$ is equal to 0.5 V.
   (c) Calculate $C_{min}$.

5. A silicon pnp bipolar transistor at $T=300 K$ has uniform dopings of $N_F = 10^{18}$ cm$^{-3}$, $N_B = 10^{15}$ cm$^{-3}$, and $N_C = 10^{15}$ cm$^{-3}$. The metallurgical base width is 1.2 $\mu$m. Let the hole diffusion coefficient $D_H = 10$ cm$^2$/sec. Assume that the minority carrier hole concentration in the base can be approximated by a linear distribution. Let $V_{BE} = 0.625$ volt. (Si : $n_i = 1.5 \times 10^{10} \text{cm}^{-3}$, $\varepsilon_{Si} = 11.8$, $\varepsilon_{s} = 8.85 \times 10^{-14} \text{F/cm}^2$, $kT/q = 0.0259 \text{eV}$) (18%)

   (a) Determine the hole diffusion current densities in the base for $V_{BC}=5$ volt, and $V_{BC}=10$ volt.
   (b) Estimate the Early Voltage.
   (c) Calculate the punch-through voltage.

6. Assume a linearly graded junction with doping distribution as $N_d - N_a = Gx$.

   The critical electric field for breakdown is $E_{crit}$. Derive the expression for the breakdown voltage. (16%)
1. (15pts) A vector field is given as \( \vec{G}(x, y, z) = \vec{a}_x (x - 3y) + \vec{a}_y (y^2 - 2z) + \vec{a}_z (z^2 - xy) \).

Find
(a) line integral from \((0, 0, 0)\) to \((1, 1, 1)\) along the straight line
(b) line integral from \((0, 0, 0)\) to \((1, 1, 1)\) along the curve \(y = x^3, \ z = x\)
(c) Does there exist a scalar field \( \Phi \) such that \( \vec{G} = \nabla \Phi \)? Is so, determine \( \Phi \)? If not, give the reason.

2. (10pts) Let \( x(t) \leftrightarrow X(j\omega), \ y(t) \leftrightarrow Y(j\omega), \) and \( z(t) \leftrightarrow Z(j\omega) \) denote Fourier transform pairs, related by

\[
Z(j\omega) = \int_{-\infty}^{\infty} z(t) e^{-j\omega t} dt, \quad z(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z(j\omega) e^{j\omega t} d\omega
\]

If \( Z(j\omega) = X(j\omega)Y(j\omega) \), express \( z(t) \) in terms of \( x(t) \) and \( y(t) \).

3. (20%) There are three questions as follows.
(a) Let \( X \) and \( Y \) be two subspaces of \( \mathbb{R}^n \). Write the definition of \( X \) and \( Y \) being orthogonal.
(b) Let \( X \) and \( Y \) be two orthogonal subspaces of \( \mathbb{R}^n \). Show that \( X \cap Y = \{0\} \).
(c) Let \( A \in \mathbb{R}^{mxn} \) with \( \text{rank}(A) = r \), and let \( \{x_1, \ldots, x_r\} \) be a basis for \( R(A^T) \), the range of \( A^T \). Is \( \{Ax_1, \ldots, Ax_r\} \) a basis for \( R(A) \)? If the answer is YES, give a proof for it. Otherwise, provide a counterexample for it.

4. (15%) For any matrix \( A \in \mathbb{R}^{mxn} \), the induced norm of it is defined as

\[
\|A\| := \sup_{x \neq 0} \frac{\|Ax\|}{\|x\|}
\]

Then choose the correct one(s) from the following four statements and give a proof for each one of your choice.
(a) \( \|A\| := \sup_{x \neq 0} \frac{\|Ax\|}{\|x\|} \).
(b) \( \|Ax\| \leq \|A\| \cdot \|x\| \), for any \( x \in \mathbb{R}^n \).
(c) \( \|A + B\| \leq \|A\| + \|B\| \), for any \( A, B \) in \( \mathbb{R}^{mxn} \).
(d) \( \|AB\| \leq \|A\| \cdot \|B\| \), for any \( A, B \) compatible for multiplication.

5. (10%) Given \( z_0 \in \mathbb{C} \), let \( \alpha := |z_0|, \ \beta := \Re(z_0), \) and \( \gamma := \Im(z_0) \).

Then solutions to the equation \( z^2 = z_0 \) are
(a) \( \pm \frac{1}{\sqrt{2}} (\sqrt{\alpha + \beta i} + \sqrt{\alpha - \beta i}) \)
(b) \( \frac{1}{2i} (\beta \pm i\gamma) \)
(c) \( \frac{1}{\sqrt{2}} (\sqrt{\beta + i\gamma}) \)
(d) \( \pm \frac{1}{\sqrt{2}} (\sqrt{\alpha + \beta i} - i\sqrt{\alpha - \beta i}) \)
(e) \( \pm \frac{1}{\sqrt{2}} (\sqrt{\alpha + \beta i} + i\sqrt{\alpha - \beta i}) \)
(f) \( \pm \frac{1}{\sqrt{2}} (\sqrt{\alpha + \beta i} - i\sqrt{\alpha - \beta i}) \)
(g) \( \frac{1}{\sqrt{2}} (\sqrt{\beta + i\gamma} \pm i\sqrt{\beta - i\gamma}) \)

Please choose your answer(s) and give all the necessary details.

6. (15%) Find the general solution of the following differential equation:

\[
y'(x + y + 1)^2
\]

7. (15%) Use the Laplace transform to solve the following initial-value problem:

\[
y'' + 4y = \sin t \quad u(t - 2\pi), \ y(0) = 1, \ y'(0) = 0.
\]
1. Please explain the following terms:
   (a) Nyquist Stability Criterion,
   (b) Characteristic Equation and its effects on the control system,
   (c) Steady-state Error and its cause on the control system,
   (d) Controllability and Observability,
   (e) Bandwidth and gain crossover frequency,
   (f) PID controller and its effects on the control system.

2. (a) Draw a state diagram for the following state equations:
   \[
   \frac{dx_1}{dt} = -2x_1(t) + 3x_2(t)
   \]
   \[
   \frac{dx_2}{dt} = -5x_1(t) - 5x_2(t) + 2r(t)
   \]

   (b) Find the characteristic equation of the system.

   (c) Find the transfer functions \( X_1(s)/R(s) \) and \( X_2(s)/R(s) \)

3. A control system with a PD controller is shown in Fig.1
   (a) Find the values of \( K_p \) and \( K_d \) so that the ramp-error constant \( K_r \) is 1000
      and the damping ratio is 0.5.
   (b) Find the values of \( K_p \) and \( K_d \) so that the ramp-error constant \( K_r \) is 1000
      and the damping ratio is 1.0

   ![Fig.1](image)
4. Give the system
\[ \frac{dx(t)}{dt} = Ax(t) + Bu(t) \quad y(t) = Cx(t) \]
where
\[ A = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix} \]
(a) Determine the state controllability and observability of the system.
(b) Let \( u(t) = -Kx(t) \), where \( K = [k_1 \quad k_2] \), where \( k_1 \) and \( k_2 \) are real constants.
Determine if and how controllability and observability of the closed-loop system are affected by the elements of \( K \).

5. A linear time-invariant system is described by the following state equations:
\[ \frac{dx(t)}{dt} = Ax(t) + Bu(t) \]
where
\[ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]
The closed-loop system is implemented by state feedback, so that \( u(t) = -Kx(t) \),
where \( K = [k_1 \quad k_2 \quad k_3] \), where \( k_1 \), \( k_2 \), and \( k_3 \) are real constants. Determine the constraints on the elements of \( K \) so that the closed-loop system is asymptotically stable.

6. The block diagram of a control system with state feedback is shown in Fig.2. Find the real feedback gains \( k_1, k_2 \) and \( k_3 \) so that
The steady-state error \( e_w \) [\( e(t) \) is the error signal] due to a step input is zero.
The complex roots of the characteristic equation are at \(-1 + j\) and \(-1 - j\).
Find the third root. Can all three roots be assigned arbitrarily while still meeting the steady-state requirement?

---

Fig. 2
考生請注意：必須寫出作答過程或說明，否則不予計分。

1. Give an example of relation $R$ on $A=\{1,2,3\}$ for each of the following property.
   (a) $R$ is not reflexive. (4%)
   (b) $R$ is not transitive. (4%)
   (c) $R$ is not anti-symmetric. (4%)
   (d) $R$ is both symmetric and anti-symmetric. (4%)
   (e) $R$ is neither symmetric nor anti-symmetric. (4%)

2. Draw a planar representation of the graph in Fig. 1 if possible; otherwise state why it is nonplanar. (10%)

![Fig. 1.](image1)

![Fig. 2.](image2)

3. (a) Find a dual graph of the graph in Fig. 2. (5%)
   (b) Find the chromatic polynomial of (a). (5%)
   (c) Find the minimum number of colors needed to paint the regions of the map in Fig. 2 from the result of (b). (5%)

4. Which of the partially ordered sets in Fig. 3 are lattices and why? (10%)

![Fig. 3.](image3)
5. Solve the recurrence relation $a_{n+2} = 4a_{n+1} - 4a_n$, $n \geq 0$, $a_0 = 1$, $a_1 = 3$. (15%)

6. For $R = \{s, t, x, y\}$, define $+$ and $\cdot$, making $R$ into a ring, by Table 1(a) for $+$ and by the partial table for $\cdot$ in Table 1(b).

<table>
<thead>
<tr>
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<th>$+$</th>
<th></th>
<th>$\cdot$</th>
<th></th>
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<td>$y$</td>
<td>$x$</td>
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</tbody>
</table>

Table 1.

(a) Using the associative and distributive laws, determine the entries for the missing spaces(?) in Table 1(b). (6%)

(b) Is this ring commutative? (3%)

(c) Does it have a unity? How about units? (6%)

(d) Is the ring an integral domain or field? (3%)

7. Use generating function to determine how many ways we can select $r$ objects from $n$ distinct objects with repetitions allowed. (12%)

Reference formula:

$$
\frac{1}{1-x} = \sum_{i=0}^{n} x^i
$$

$$
\frac{1}{1-x} = \sum_{i=0}^{n} \binom{n+i-1}{i} x^i
$$

$$
\frac{1}{1-x^2} = \sum_{i=0}^{n} \binom{n+i-1}{i} x^i
$$
1. (15 points) Write a pseudocode function for each of the following cases to determine whether a string is in the language $L$, where $L$

(a) $L = \{ w \mid w \text{ contains equal numbers of } A's \text{ and } B's \}$.

(b) $L = \{ w \mid w \text{ is of the form } A^n B^n \text{ for some } n \geq 0 \}$.

(c) $L = \{ w\$w' \mid w \text{ is a possibly empty string of characters other than } \$ \}$, where $w'$ is the reverse of $w$.

2. (15 points) Consider a sequential search of $N$ data items.

(a) If the data items are sorted into ascending order, how can you determine that your desired item is not in the data collection without always making $N$ comparisons?

(b) What is the order of the sequential search algorithm when the desired item is not in the data collection? Do this for both sorted and unsorted data, and consider the best, average, and worst cases.

(c) Show that if the sequential search algorithm finds the desired item in the data collection, the algorithm's order does not depend on whether or not the data items are sorted.

3. (10 points) Answer the following questions about binary trees.

(a) Starting with an empty binary search tree, in what order should you insert items to get the binary search tree shown in Figure 1?

(b) What complete binary tree does the array in Figure 2 represent? Please draw it.

4. (10 points) Answer the following questions about heaps.

(a) Does the array in Figure 2 represent a heap?

(b) Consider the heap in Figure 3. Draw the heap after you insert 12 and then remove 12.

5. (10 points) If $h(x) = x \mod 7$ and separate chaining resolves collisions, what does the hash table look like after the following insertions occur: 8, 10, 24, 15, 32, 17? Assume that each table item contains only a search key.

6. (10 points) Answer the following questions about graphs.

(a) Use the depth-first strategy and the breadth-first strategy to traverse the graph in Figure 4, beginning with vertex 0. List the vertices in the order in which each traversal visits them.

(b) Draw the minimum spanning tree whose root is vertex 0 for the graph in Figure 5.
7. (15 points) Given a list of integers 2, 7, 10, 25, 33, 44, 56, 78, 101, 150, 200, 207, 350, 400, and 450, which integers would be interrogated by the binary search
(a) when searching for 44?
(b) when searching for 500?
(c) what is the maximum number of comparisons that must be made when applying the binary search to a list of 2000 entries?

8. (15 points) Suppose we have a list of integers 46, 41, 37, 68, 90, 53, 72, 84, 65, and 59. Describe a hash function to store these integers such that no collisions will occur.

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Figure 1: Figure for Problem 3(a).

Figure 2: Figure for Problem 3(b) and Problem 4(a).
Figure 3: Figure for Problem 4(b).

Figure 4: Figure for Problem 6(a).

Figure 5: Figure for Problem 6(b).
1. (10%) Number representation
   (a) (5%) Convert $9AD_{16}$ to its octal representation.
   (b) (5%) Represent $-576_{16}$ in 16-bit 2's complement representation.

2. (10%) Reduce the logic expression of $Y = abc + a'd'b + ab'c'd + a'b'c'd' + a'b'd' + a'cd'$ and draw its minimized 2-level logic circuit.

3. (5%) Given a Karnaugh map of output $Y$, write its minimized logic equation.

<table>
<thead>
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<th>cd</th>
<th>00 01 11 10</th>
</tr>
</thead>
<tbody>
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<td>00</td>
<td>0 1 1 d</td>
</tr>
<tr>
<td>01</td>
<td>0 1 1 0</td>
</tr>
<tr>
<td>11</td>
<td>d 1 0 1</td>
</tr>
<tr>
<td>10</td>
<td>1 0 d d</td>
</tr>
</tbody>
</table>

4. (5%) Given a $3 \times 8$ decoder module, design a 1-bit full adder with it.

5. (15%) Design the 7-segment display conversion combinational circuit.
   (a) (5%) Write the truth table for BCD input $b_3, b_2, b_1, b_0$ and 7-segment output $a, b, c, d, e, f, g$.
   (b) (10%) Design the 7-segment display with a 4-bit decoder, a 4x7 ROM module, and wire OR gates. You need to show ROM internal connections.

6. (15%) Design an adder/subtractor for 4-bit 2's complement numbers with 4 1-bit full adder modules, multiplexors, inverters.
   - When $OP = 0$, it performs addition.
   - When $OP = 1$, it performs subtraction.
7. (10%) Design an up/down counter between 0 and 15. (You can use 4 T flip-flops.)
   - When \( OP = 0 \), it performs up-counting at each clock cycle.
   - When \( OP = 1 \), it performs down-counting at each clock cycle.

8. (10%) Design a 4-bit parallel-in serial-out circuit.
   - When \( \text{parallel}_{in} = 1 \), \( A3, A2, A1, \) and \( A0 \) are transferred into 4 D flip-flops \( D3, D2, D1, \) and \( D0 \) in parallel at the clock period.
   - When \( \text{parallel}_{in} = 0 \), \( D3, D2, D1, \) and \( D0 \) are shifted outside in serial at each clock period.

9. (20%) Design a sequence detector which recognizes "1101" on the input bit stream \( I \).
   - When the pattern is recognized, it produces '1' on output \( R \).
   - Otherwise, it produces '0' on output \( R \).
   (a) (10%) Draw its state transition diagram.
   (b) (10%) Draw its logic circuit design.
I. (25%) Basic Computer Architectures

(1) (10%) Consider a hypothetical microprocessor generating a 16-bit address (for example, assume that the program counter and the address register are 16 bits wide) and having a 16-bit data bus.

(a) (5%) What is the maximum memory address space that the processor can access directly if it is connected to an 8-bit memory?

(b) (5%) If an input and an output instruction can specify an 8-bit I/O port number, how many 16-bit I/O ports can the microprocessor support?

(2) (15%) Consider a hypothetical 32-bit microprocessor having 32-bit instructions composed of two fields: the first byte contains the OP code and the remainder the immediate operand or an operand address. Assume that the local address bus is 32 bits and the local data bus is 16 bits. No time multiplexing between the address and data buses.

(a) (5%) What is the maximum directly addressable memory capacity (in bytes)?

(b) (5%) What is the minimum bit numbers required for the program counter?

(c) (5%) Assume the direct addressing mode is applied, how many address and data bus cycles required to fetch an instruction and its corresponding operand or data from memory?

II. (25%) Computer Arithmetic Architectures

(1) (10%) Consider two packed decimal numbers in which two unsigned numbers are added together. If each number consists of N digits, then there are 4N bits in each number. The two numbers are to be added using a binary adder with some correction rules. Suggest a simple rule for correcting the final result and then perform addition in this fashion on the two numbers 1698 and 1786.

(2) (15%) Perform the following three Intel 386/486 instructions,

\[
\begin{array}{ll}
\text{MOV} & \text{AX} \ 0248H \\
\text{MOV} & \text{BX} \ 0564H \\
\text{CMP} & \text{AX} \ \text{BX}
\end{array}
\]

and list the Carry Flag(CF), Overflow Flag(OF), Parity Flag(PF), Sign Flag(SF), and Zero Flag(ZF).
III. (25%) System Performance and Cache Memory

(1) (10%) A 600-MHz Pentium-III processor is used to execute a benchmark program with the following instruction mix and clock cycle time:

- Integer Arithmetic: Instruction Count = 60000 Clock Cycle Count = 1
- Data Transfer: Instruction Count = 35000 Clock Cycle Count = 2
- Floating Point: Instruction Count = 55000 Clock Cycle Count = 4
- Control Transfer: Instruction Count = 30000 Clock Cycle Count = 3

(a) (5%) Calculate the effective CPI (Cycles Per Instruction).
(b) (5%) Calculate the MIPS (Million Instructions Per Second).

(2) (15%) Consider a cache (M1) and a main memory (M2) hierarchy, in which M1 = 32 K words, M2 = 2 M words, and cache block size = 8 words.

(a) (5%) Show the physical address format of Direct Mapping (Show how many Tag, Block, and Word bits).
(b) (5%) Show the physical address format of 4-way Set Associate Mapping (Show how many Tag, Set, and Word bits).
(c) (5%) Calculate the MIPS, if we assume the hit ratio = 0.9, the average instruction execution time after cache hit = 10 nsec, and the memory access time for one block = 100 nsec when cache miss.

IV. (25%) Pipeline and Multiprocessor Architectures

(1) (10%) A BBN Butterfly Switch Network consists of 8x8 crossbar switch modules and eight-way shuffle multistage interconnection.

(a) (5%) Calculate how many 8x8 switch modules will be needed for a 512x512 Butterfly Switch Network?
(b) (5%) Assume a binary address, X = (x8 x7 x6 x5 x4 x3 x2 x1 x0). What will be the corresponding address after the operation of the eight-way shuffle.

(2) (15%) Consider the execution of the following code fragment of an assemble program, where R1-R6 represent the CPU internal registers and A, B represent two memory locations, respectively. Assume a 4-stage pipeline consisting of IF (Instruction Fetch), ID (Instruction Decode), IE (Instruction Execute) and RS (Result Store), and the branch condition cannot be resolved until at the RS stage.

I1: Load R1 A
I2: Dec R3 1
I3: BrZero R3 I5 / Jump to I5 if Zero Flag = 1/
I4: Add R2 R4
I5: Sub R5 R6
I6: Store R5 B

(a) (5%) How many delayed branch cycles will be generated?
(b) (5%) Reorder the instruction sequences, and identify the two useful delay instructions.
(c) (5%) How many delayed branch cycles will be generated after the instruction recording?
1. 描述 Thevenin and Norton equivalents, 並說明如何求得 Thevenin equivalent voltage source, Thevenin equivalent resistance and Norton equivalent current source? (10%)

2. 描述 voltage follower 的功能 (10%)

3. 何謂 virtual short circuit 以及 virtual open circuit? (10%)

4. 在電路學中，Laplace transform 及 Fourier transform 有什麼功能？二者之功能又有什麼區別？ (10%)

5. 在電路學中，微分方程式的 forced solution 及 natural solution 所代表的物理意義是什麼？ (10%)

6. 描述 phasor 的功能。在電路學中可以使用 phasor 的條件是什麼？ (10%)

7. 電力系統的功率因素(power factor) 常常不高，原因何在？如何補救？ (10%)

8. Find the three resistive currents shown in Fig. 8 (15%)

9. Find v(t), t > 0 for i(0-) = 1 A and v(0-) = 4 V (15%)

Fig. 8

Fig. 9
1) A 3Φ, 60Hz, 345KV Transmission line with the arrangement of conductors of a parallel circuit as shown in Fig.1. The GMR and outside diameter of the conductors are 0.0229 ft and 0.680 inch respectively. (a) Find the inductance in ohms per kilometer per phase and the per unit value with 100MVA as the base. (b) Find the capacitive susceptance in siemens per kilometer per phase. (20%)

2) Calculate the power flow and line power loss for a transmission line as shown in Fig.2.

\[ V_s < \theta_s \quad \frac{\psi r}{\theta r} \quad \frac{\psi s}{\theta s} \]

Fig. 2

(15%)

3) For the power system as shown in Fig.3, (a) Determine the admittance matrix. (b) Solve the power flow analysis by Gauss method for one iteration only. (25%)

Fig. 3
4) In Fig. 4, a generator is connected to an infinite bus and deliver 1.2 PU power. The terminal voltages of bus 1 and 2 are 1.0 PU. (a) Solve the power angle of the generator at the steady state. (b) When a solid 3φ fault occurs at the middle point P of one of the circuit, derive the equation to solve the critical clearing angle. (Illustrate by the power angle curve for the pre-fault, fault and post-fault periods) 

\[ X_d = 0.2, 0.1 \]

\[ \text{Fig. 4} \]

5) Describe various types of power quality problem and the conventional methodologies to alleviate the problem. (15%)
1. (a) Write down the integral form of time-harmonic Maxwell’s equations.
   (b) Starting from these equations, derive all the boundary conditions at the interface between two different materials.
   (20%)

2. A uniform plane wave in air is incident on a lossless dielectric material at a 45° angle, as shown in Fig. P2. The transmitted wave propagates in a 30° direction with respect to the normal. The frequency is 300 MHz.
   (a) Find $\varepsilon_2$ in terms of $\varepsilon_0$.
   (b) Find the reflection coefficient and transmission coefficient.
   (c) Obtain the mathematical expressions for the incident, reflected and transmitted fields.
   (20%)

3. A cylindrical capacitor carries a total charge of $+Q$ and $-Q$ on the inner and outer conductors, respectively. The capacitor is not connected to the ground or to any voltage source. A dielectric material with dielectric constant $\varepsilon_1$ and inner and outer diameters matching the dimensions of the cylindrical capacitor, is placed x meters inside the capacitor as shown in Fig. P3.
   (a) Find the electric field in the capacitor in terms of $Q$.
   (b) Find the voltage between the two conductors.
   (c) Find the force acting on the dielectric material.
   (20%)
4. Three infinitely long parallel wires each carry 10 A of current in the z-direction, as shown in Fig. P4. Find the force per unit length acting on the Number 3 wire due to the magnetic fields produced by the other two wires.  

![Diagram of three parallel wires with magnetic fields](image)

Fig. P4

5. Briefly answer the following questions. (5% each)
   
   (a) What is the meaning of the polarization of a plane wave?
   
   (b) What is skin effect? Discuss your knowledge about skin effect.
   
   (c) What is the Hall effect?
   
   (d) What is a Smith chart and why is it useful in making transmission line calculations?
計分說明(並非考題)
本科計分將採用一種新創的”市場需求調整計分法”，即計分高低(價值)與
獲解率(市場需求率之倒數，愈易獲解，市場上需求愈低)成反比，所以考生除了
要把握多數人會的題目外，還要著重於自己獨特的能力，即別人不容易會的問題。
在入學徵才錄取率不高的情況下，希望考生能發揮出自己卓越的特點。

你的得分\(S'_1, S'_2, ..., S'_6\)公式計算如下：令\(R'_1, R'_2, ..., R'_6\)為六小題各自原始分
數給分範圍，\(R'_1, R'_2, ..., R'_6\)為依市場需求調整後的分數給分範圍，\(M_1, M_2, ..., M_6\)
為三小題各自的原始平均得分，\(S_1, S_2, ..., S_6\)為六小題各自原始得分：

则\(S'_i = S_i \times \frac{R'_i}{R_i}\)

在\(M_1, M_2, ..., M_6\)的各小題平均分數算出後，

\(R'_1 + R'_2 + ... + R'_6 = 100\) 且 \(R'_1 : R'_2 : ... : R'_6 = \frac{R_1}{M_1} : \frac{R_2}{M_2} : ... : \frac{R_6}{M_6}\)

得到\(R'_1, R'_2, ..., R'_6\)。

注意\(R'_1, R'_2, ..., R'_6\)的原始範圍可以任意設定，藉著讓考生易於依循，我仍可以做
一次不必要的設定：\(R_1 = R_2 = ... = R_6 = \frac{100}{6}\)。

1. (A)
   (a) What is the dimension of the set of complex numbers (if it is a vector space)
       over the field of real numbers?
   (b) What is the dimension of the set of complex numbers (if it is a vector space)
       over the field of complex numbers?
   (c) What is the dimension of the set of real numbers (if it is a vector space) over the
       field of complex numbers?

   (B)
   Prove your answer of the dimension if the referred set is indeed a vector space, or
disprove it if the set is not a vector space.

2. A linear system with its input \(x(t)\) and output \(y(t)\) is described by the following
differential equation: \(y'(t) + 2y(t) = x(t)\). This linear system can be regarded as a
linear operator mapping the input \(x(t)\) to the output \(y(t)\). Please find the eigenvectors
for this operator.
3. Reading Comprehension: (閱讀測驗)

Suppose that $V$ and $W$ are finite-dimensional vector spaces with ordered bases $\beta = \{x_1, \ldots, x_m\}$ and $\gamma = \{y_1, \ldots, y_n\}$, respectively. Let $T : V \rightarrow W$ be linear. Then there exist unique scalars $a_{ij}$ belonging to the scalar field $F(i = 1, \ldots, m$ and $j = 1, \ldots, n)$ such that $T(x_i) = a_{ij}y_j$ for $1 \leq i \leq m$. Using the notation above, we call the $m \times n$ matrix $A$ defined by $A_{ij} = a_{ij}$ the matrix that represents $T$ in the ordered bases $\beta$ and $\gamma$ and will write $A = [T]_\beta^\gamma$. Furthermore, we denote by $L_A$ the mapping $L_A : F^n \rightarrow F_m$ defined by $L_A(x) = Ax$ (the matrix product of $A$ and $x$) for each column vector $x \in F^n$. We call $L_A$ a left-multiplication transformation. It can be proved that $L_A$ is linear. We have the following properties:

1. $[L_A]_\beta^\gamma = A$.
2. $L_A = L_B$ if and only if $A = B$, where $B$ is a $m \times n$ matrix.
3. $L_{AB} = L_A + L_B$ and $L_{aA} = aL_A$ for all $a \in F$.
4. There exists a unique $m \times n$ matrix $C$ such that $T = L_C$. In fact $C = [T]_\beta^\gamma$, where $T$ is defined as above.
5. If $E$ is an $n \times p$ matrix, then $L_{AE} = L_AL_E$.
6. If $m = n$, then $L_{In} = I^n$.

(A) Which of the above statements (1) to (6) are regarding the following properties (a) linearity (b) well defined function (c) onto function (d) one-to-one function (e) invertible function?

(B) Which conceptual function (transformation) are the above properties associated to?

4. Three events $E_1$, $E_2$, and $E_3$, defined on the same space, have probabilities $P(E_1) = P(E_2) = P(E_3) = 0.25$. Let $E_0$ be the event that one or more of the events $E_1$, $E_2$, $E_3$ occurs.

(A) Find $P(E_0)$ when the events are statistically independent.

(B) Find the maximum values that $P(E_0)$ can assume and explain the physical meanings for this situation.

5. A random variable $X$ is linearly related to another random variable $Y$ by the relation $Y = -3X + 5$. Can we find the correlation coefficient of $X$ and $Y$ when we do not know the mean and variance of $X$ and $Y$? Compute the correlation coefficient if the answer is yes or prove it if the answer is no.

6. Describe all the relations which you know between linear algebra and probability.
一. Explain in detail what is Additive White Gaussian Noise (AWGN)? (20分)

二. What is Wiener Filter? What is Matched Filter? Explain their objectives, their principles, and their implementations. (20分)

三. What is Linear Time-Invariant Lumped System? (10分)
   Is the AM modulator linear system? Why?
   Is it a time-invariant system? Why?

四. What is Gram-Schmidt procedure? (10分)
   Explain its objectives, principles, and give one example by 3 signals.

五. In general, the music program is transmitted by FM stations instead of AM stations, explain clearly what reasons it's for? (10分)

六. Explain the difference among Frequency Division Multiple Access (FDMA), Time-Division Multiple Access (TDMA), and Code Division Multiple Access (CDMA). Why do you think future mobile phones utilize wide-band CDMA to communicate each other? How and where the "wide-band" comes from? (10分)
七. Explain what are the differences among positively correlated, negative correlated and un-correlated random signals? Give your example on 2-Dimensional space. How can we use these properties to classify signals? (10 marks)

八. When you study binary hypothesis problem, the professor always wrote down the following:

Cost function, \( C_{ij} \)

Conditional probability density function, \( p(r|H_i) \)

Risk, \( R \)

Assuming \( H_0 \rightarrow \) not send any signal \( \quad H_i \rightarrow \) send signal with mean \( \frac{A}{\sqrt{2}} \) volts

Noise is \( AGWN \)

Derive the corresponding Bayesian Decision Rule, Maximum A Posteriori Decision Rule, and Maximum Likelihood Decision Rule.

(10 marks)