Basic Circuit Analysis (20% each)

[Problem 1] For the following circuit, a current-control voltage source is attached between node 3 and 4. Try to find all the node voltages.

[Diagram]

[Problem 2] For the following OPAMP, $R_a=1k\Omega$, $R_b=10k\Omega$, $R_i=10k\Omega$, $R_o=100\Omega$.
With $A=100$, Find
(a) $G=V_o/V_s$, $R_i=V_s/I_s$
(a) The close-loop output resistance $R_o$

[Diagram]
Problem 3] A 3-phase 3-wire 240V network ACB is connected to a △ load as shown in the figure below. Try to find (1) the phase current (2) the line current, and (3) the plot in polar axis.

\[ Z_{AB} = 10 \angle 30^\circ \ \Omega \]
\[ Z_{BC} = 20 \angle 90^\circ \ \Omega \]
\[ Z_{CA} = 15 \angle -45^\circ \ \Omega \]

Problem 4] With a parallel RLC circuit, Try to find
(1) The unity-power-factor frequency.
(2) the voltages on the L and C elements

Problem 5]

Find the thevenin and Norton EQ ckt
1. For the power system in Fig. 1, (all units are in p.u with 100 MVA base), Bus 1 (swing bus) and Bus 2 are connected by a transformer with impedance of j 0.25 p.u and tap ratio 1:1.05. (25%)

   1) Find the equivalent circuit of the regulating transformer.
   2) Solve the voltage at Bus 2 by Newton Raphson method for one iteration. The initial voltage of Bus 2 is $1.0 \angle 0^\circ$. The voltage level of swing bus $V_s = 1.0$

   ![Fig 1](image)

2. With load flow analysis for the system in Fig. 2 (all units in p.u), the bus voltage has been solved as $V_2 = 0.98183 \angle 3.5035^\circ$ and $V_3 = 1.00125 \angle 2.8624^\circ$. Solve the power delivered by swing bus and total system power loss. (15%)

   ![Fig 2](image)

3. A 69KV, three phase transmission line is 40 km long with per phase series impedance of $(0.125 + j 0.50) \Omega$ per km. Determine the voltage regulation and the transmission efficiency when the line delivers 100 MVA with power factor 0.8 lagging at 66KV. (20%)

4. (a) Describe the critical clearing time and how the equal area criterion is applied for power system stability analysis. (10%)

   (b) Describe the unit commitment, economic dispatch control for power system operation. (5%)
5. For a power system in Fig. 3, the neutral of each generator is grounded with a reactor of 0.1 p.u on a 100 MVA base. The system data are shown as the tables below. What's the fault current for a double line to ground fault at Bus 2 through a fault impedance $Z_f=48.4\Omega$ (25%)

![Fig 3]

<table>
<thead>
<tr>
<th>Item</th>
<th>Capacity Rating (MVA)</th>
<th>Voltage Rating (kV)</th>
<th>$X^1$</th>
<th>$X^2$</th>
<th>$X^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_1$</td>
<td>100</td>
<td>20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$G_2$</td>
<td>150</td>
<td>20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$T_1$</td>
<td>100</td>
<td>20/220</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$T_2$</td>
<td>200</td>
<td>20/220</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line</th>
<th>$X^1$ (Ω)</th>
<th>$X^2$ (Ω)</th>
<th>$X^0$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{12}$</td>
<td>60</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>$L_{13}$</td>
<td>60</td>
<td>60</td>
<td>150</td>
</tr>
<tr>
<td>$L_{23}$</td>
<td>60</td>
<td>60</td>
<td>150</td>
</tr>
</tbody>
</table>
1. A spherical capacitor is filled with a dielectric material of $\varepsilon_1$ in half of the space and with another material $\varepsilon_2$ in the remaining space, as shown in Fig. P1.

(10%) (a) Find the potential distribution in the region $a < r < b$ assuming the potential at $r = a$ is $V_0$ and at $r = b$ is 0.

(5 %) (b) Find the electric field in the region $a < r < b$.

(5 %) (c) Find the electric flux density in the region $a < r < b$.

![Fig. P1.](image)

2. (20%) A rectangular loop is located in the vicinity of an infinitely long wire carrying a direct current $I$ as shown in Fig. P2. Center of the rectangular loop is located at $x = d + a/2$. Find the voltage induced in the loop if it is rotating about the axis parallel to the z-axis with an angular frequency $\omega$.

![Fig. P2.](image)

3. (20%) A light ray is obliquely incident from one side of a prism at an angle $\theta_i$ as shown in Fig P3. The angle $\theta_i$ is adjusted to a specific angle $\psi_c$ such that the incident angle on the other side of the prism is equal to the critical angle of incidence $\theta_c$. Derive the expression for refractive index of the prism in terms of the angles $\lambda$ and $\psi_c$. 
4. (10%)(a) What is a microstrip line? How to control the characteristic impedance of a microstrip line?
   (10%)(b) A 100 Ω microstrip line has an effective dielectric constant of 1.65. Find the shortest open-circuited length of this line that appears at its input as a capacitor of 5 pF at 2.5 GHz. Also find the shortest short-circuited length for an inductor of 5 nH at 2.5 GHz.

5. A standard air-filled K-band rectangular waveguide has dimensions $a = 1.07$ cm and $b = 0.43$ cm.
   (10%)(a) Determine the first two propagating modes and their cutoff frequencies.
   (10%)(b) If this rectangular waveguide is made from perfect conductor and filled with a dielectric material having $\varepsilon_r = 2.2$ and $\tan\delta = 0.002$, compute the TE$_{10}$ mode attenuation in dB/m at 20 GHz.
計分說明：

本題計分將採用一種新創的“市場需求調整計分法”，即計分高低（價值）與獲解率（市場需求率之倒數，愈易獲解，市場上需求愈低）成反比。所以考生除了要把握多數人會的題目外，更要著重於自己獨特的能力，即別人才不容易會的問題。在入學徵才錄取率不高的情況下，希望考生能發揮出自己卓越的特點。

你的得分（$S_1, S_2, S_3, \ldots$）公式計算如下：令$R_1, R_2, R_3, \ldots$為小題各自的原始分數，$R_1', R_2', R_3', \ldots$為依照市場需求調整後的分數給分範圍，$M_1, M_2, M_3, \ldots$為各小題各自的原始平均得分，$S_1, S_2, S_3, \ldots$為小題各自的原始得分：

则$S_i = S_i \times \frac{R_i'}{R_i}$

而$R_1', R_2', R_3', \ldots$的計算，在$M_1, M_2, M_3, \ldots$的各小題平均分數算後，

根據$R_1': R_2': R_3': \ldots = \frac{R_1}{M_1} : \frac{R_2}{M_2} : \frac{R_3}{M_3} \ldots$得到，且$R_1' + R_2' + R_3' + \ldots = 100$

注意$R_1, R_2, R_3, \ldots$的原始範圍可以任意設定，為著讓考生易於掌握，所以我仍可以做一次不必要的設定：$R_1 = 10, R_2 = 20, \ldots = \frac{100}{k}$

不要怕會（考卷附圖，依圖示為，不必計算）

1. The following plot demonstrates the magnitude response of VSB filter.

![Plot](attachment:plot.png)

(a) Why the VSB (Vestigial Sideband Modulation) are demanded in communications such as TV applications?

(b) Draw the plot of $H_v(f - f_0)$ (only $H_v(f - f_0)$)

(c) Derive the requirement for the response of VSB that

$H_v(f - f_0) + H_v(f + f_0) = 1.$

You must start by the modulated signal denoted as the following

$\Phi_m(\omega) = \frac{1}{2} F(\omega - \omega_v) + \frac{1}{2} F(\omega + \omega_v) \cdot H_v(\omega)$

Where $\omega = 2 \pi f.$
2. (a) What is the function of the following block diagram.

(b) Prove your answer by appropriate mathematics.

3. The following diagram demonstrates some famous line codes.

(a) Which one has the best bandwidth performance? Please explain.

(b) Which one has the best error probability performance? Please explain.

You don’t need to give the names of codes. Concentrate on explanation.
4. The following diagram demonstrates the ideal basic pulse shape of an ideal Nquist channel.

(a) What is the idea of basic pulse shape?

(b) What should be the basic pulse shape if the channel response (transfer function) is triangle instead of rectangle? Please explain.

5. (a) What are the functions of the following block diagram?
   What is the function of the block of on-off level encoder?
   (b) Give the outlines for computing the bit error rate in terms of the $E_b$ and $N_0$ as commonly defined for signal and noise.

\[
\phi_1(t) = \sqrt{2T_b} \cos(2\pi f_0 t)
\]

\[
\phi_2(t) = \sqrt{2T_b} \cos(2\pi f_1 t)
\]

(a) 

(b) 

Choose 1 if $y > 0$

Choose 0 if $y < 0$

Threshold = 0
1. What is singular value decomposition (SVD)? Why do we need it? Explain your concepts in detail and then compute the SVD of the following matrix:

\[
A = \begin{bmatrix}
1 & 1 \\
1 & 1 \\
0 & 0
\end{bmatrix}
\]

2. What are the eigenvalues, eigenvectors and the eigenspaces of a matrix? Why do we study them? Explain your concepts in detail and then compute the eigenvalues, eigenvectors and the eigenspaces of the following matrix:

\[
A = \begin{bmatrix}
2 & -3 & 1 \\
1 & -2 & 1 \\
1 & -3 & 2
\end{bmatrix}
\]

3. Find the \(N(A)\), \(R(A^T)\), \(N(A^T)\), and \(R(A)\) of the following matrix:

(where \(N\), \(R\) and \(T\) denote null space, range space and transpose of a matrix.)

\[
A = \begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 1 \\
1 & 3 & 4
\end{bmatrix}
\]

4. Compute the Gram-Schmidt QR factorization of the matrix:

\[
A = \begin{bmatrix}
1 & -2 & -1 \\
2 & 0 & 1 \\
2 & -4 & 2 \\
4 & 0 & 0
\end{bmatrix}
\]

5. Let \(A\) be a symmetric tridiagonal matrix (i.e., \(A\) is symmetric and \(a_{ij} = 0\) whenever \(|i - j| > 1\). Let \(B\) be the matrix formed from \(A\) by deleting the first two rows and columns. Show that (where \(\det\) and \(M\) denote determinant and sub-matrix,)

\[
\det(A) = a_{11} \det(M_{11}) - (a_{12})^2 \det(B)
\]
1. (15%) Binary Search.
   (a) (10%) Given the following list, show the steps of locating the target '357' by binary search:

   10 25 78 97 105 131 189 207 210 241 232 357 366 465 501 567 622 793

   In particular, show at least the 'top' and 'bottom' indices in each step.

   (b) (5%) For this particular example, how much faster (in terms of percentage in reduction of the number of comparisons made) does binary search perform compared to sequential search?

2. (10%) Sorting
   You are told that a list of 10,000 words is already in order but you wish to check it to make sure and sort any words found out of order. Which of the following sorting algorithms would you choose: insertion sort, selection sort, merge sort, quick sort or bubblesort? Explain your answer clearly.

3. (25%) Tree / Binary (Search) Tree
   (a) (5%) How many edges are there in a tree if the tree has n vertices? Explain your answer clearly.

   (b) (5%) A 2-tree (or extended binary tree) is useful in analyzing the efficiency of a searching algorithm. If the height of a 2-tree is 3, what are the largest and the smallest numbers of vertices that can be in a tree? Use one example each to demonstrate your answers.

   (c) (10%) An AVL tree is a binary search tree in which the heights of the left and right subtrees of the root differ by at most 1 and in which the left and right subtrees are again AVL trees. As in Figure 1, after deleting node p, rearrange the nodes such that the resulting tree is still an AVL tree. Show the steps of such a rearrangement. Try to make as few moves as possible.

   (d) (5%) Construct a binary tree such that its preorder sequence is: ABCDEFGHI and its inorder sequence is: BCAEDGHI.

   Figure 1

4. (10%) Graph
   Given a directed graph (as in Figure 2), find its shortest paths from vertex a to every other vertex in the graph. Show the running steps clearly.

   Figure 2
5. (20%) List
(a) (5%) Given a linked list (as in Figure 3), write pseudocode for inserting a node (pointed by pointer q) after the node pointed by pointer p. Use null to represent the value of a pointer that points to nothing.
(b) (5%) Continued from (a), write pseudocode for deleting the node after the node pointed by pointer p.
(c) (10%) Consider using circular array to implement a queue. The ‘front’ and ‘rear’ indices are used to indicate the beginning and the end of the queue. Describe the problem of distinguishing the condition of an empty queue from that of a full queue. Present one approach that solves the problem.

![Figure 3](image)

6. (20%) Hash Table
(a) (5%) Given the following pairs of elements and associated key values:

('Jim', 234) ('Tom', 159) ('Amy', 933) ('Ron', 659) ('Ann', 054) ('Kim', 131)

If we put them into a hash table one by one using the hash function:

\[ H = \text{(sum of all the digits in the key)} \pmod{11} \]

What does the resulting hash table look like? Suppose the size of the hash table is 11 and collision is resolved by chaining.

(b) (10%) Design a new hash function such that the elements are more evenly distributed than in (a). Show the resulting table.

(c) (5%) There are two lists. One list contains 10,000 elements and the other contains 25 elements. By sequential search, how much longer does it take, in average (in terms of number of comparisons), to retrieve an element from the former list than from the latter. If we put the elements of the former list into Hash Table 1 of size 40,000, and the elements of the latter into Hash Table 2 of size 100. Suppose the elements are evenly distributed in both hash tables. How much longer does it take, in average, to retrieve an element from Hash Table 1 than from Hash Table 2?
[Problem 1] Given the basic gates such as NOT, AND, OR, NAND, NOR and XOR, you are asked to answer the following questions.

(a) Implements a 4-bit carry-lookahead adder, named CLA4. (10%)  
(b) Uses the CLA4, designed in the problem 1.a, to implement a two-digit BCD adder. (15%)

[Problem 2] Prove the validity of the expression Overflow = C_n XOR C_{n-1} for addition of n-bit signed numbers.
(Hint: The C_{n-1} is the carry-out from the MSB position and the C_n is the carry-out from the sign-bit position) (15%)

[Problem 3]  
(a) Show a circuit that implements the gated SR latch using NAND gates only. (5%)  
(b) Describe the truth table of the gated SR latch, designed at 3.a, and express what conditions can be an unpredictable behavior. (10%)  

[Problem 4] Figure 1 shows a state table for a particular FSM. Minimize the number of states. (15%)

<table>
<thead>
<tr>
<th>Present state</th>
<th>Next State</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w = 0</td>
<td>w = 1</td>
</tr>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>D</td>
<td>F</td>
</tr>
<tr>
<td>C</td>
<td>F</td>
<td>E</td>
</tr>
<tr>
<td>D</td>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>E</td>
<td>F</td>
<td>C</td>
</tr>
<tr>
<td>F</td>
<td>E</td>
<td>D</td>
</tr>
<tr>
<td>G</td>
<td>F</td>
<td>G</td>
</tr>
</tbody>
</table>

Figure 1

[Problem 5]  
(a) Write the code for a negative-edge-triggered D flip-flop with asynchronous reset in VHDL or Verilog HDL (10%)  
(b) In VHDL or Verilog HDL, write the code for the traffic light controller, whose lighting sequences are as shown in Figure 2. After the red lamp lights 2 cycles, the green lamp lights 3 cycles. After the green lamp lights, the yellow lamp lights 1 cycle. (20%)

Figure 2
1. (15%) In a company of 90 employees, there are 15 managers, 44 persons in engineering division, 35 female employees. 7 managers are in engineering division. 14 female employees are in engineering division. 20 employees are male, not managers, and not in engineering division. Find the number of managers who are male and not in engineering division.

2. (15%) In an 8 x 8 chessboard, compute the number of all possible L-shape blocks with width W=1, side length L1=4, and side length L2=3 in any multiple of 90° angles at any positions within the chessboard.

3. (15%) Given an unfair 6-face die, on each toss of the die, the probabilities of getting face values 1, 2, 3, 4, 5, and 6 are 0.2, 0.2, 0.2, 0.2, 0.1, and 0.1, respectively. When we toss the die n times, calculate the probability of getting a total face value = 6n - 4. (You need only write the formula and explain the cases. You need not perform the calculation of summing up the values.)

4. (15%) Prove that the transitive closure of a symmetric relation is a symmetric relation.

5. In a complete graph of n vertices,
   (a) (10%) Calculate the number of Hamiltonian circuits exist in the graph.
   (b) (10%) Calculate the number of all possible spanning trees in the graph.
   (c) (10%) If n is an even number, in order to divide the graph into two equal size subgraphs, what is the size of such a cut set in the graph?
   (d) (10%) Calculate the total number of all possible cut sets for equal-size bi-partition of the graph as in subproblem (c) given that n is even.
1. Let $X$ be a random variable, nonnegative or not. (a) For any real $t > 0$, show the Chernoff inequality: (10%)

$$P( X \geq a ) \leq e^{-t a} M_X (t)$$

where $M_X (t)$ is defined in (P-1). Next, if $X_1, X_2, ..., X_n$ are considered to be independent random variables with moment generating functions, $M_{X_1} (t), M_{X_2} (t), ..., M_{X_n} (t)$, respectively, where $M_{X_i} (t)$ is defined as

$$M_{X_i} (t) = E[e^{tX_i}] = \int_{-\infty}^{\infty} e^{tx_i} f_{X_i} (x_i) dx_i \quad (P-1)$$

(b) If we let the random variable $Y = X_1 + X_2 + ... + X_n$, please find $M_Y (t)$, in terms of $M_{X_i} (t), i = 1, ..., n$, (10%).

(c) Now, assume that $X_i$ is with Gaussian distribution (or Normal variable), and the probability density function (p.d.f.) is given by

$$f_{X_i} (x_i) = \frac{1}{\sqrt{2\pi \sigma_i^2}} e^{-\frac{(x_i - \mu_i)^2}{2\sigma_i^2}},$$

where the mean and variance are defined as $\mu_{X_i}$ and $\sigma_{X_i}$, respectively. For $N=2$, please compute $M_Y (t)$. (10%).

2. (Sum of Poisson random variables) Let $X$ and $Y$ are random variables with Poisson PMF as follows:

$$P_X (k) = \frac{1}{k!} e^{-\lambda_1} \lambda_1^k$$

and

$$P_Y (j) = \frac{1}{j!} e^{-\lambda_2} \lambda_2^j$$

(a) Find $P[Z = m]$ where $Z = X + Y$ (10%) (b) Compute $P[Z \leq 5]$ for $\lambda_1 = 2$ and $\lambda_2 = 3$. (10%) (c) Three uncorrelated random variables $X_1, X_2,$ and $X_3$ have means $\mu_1 = 1$, $\mu_2 = -3$, and $\mu_3 = 1.5$ and second order moments $E[X_1^2] = 2.5$, $E[X_2^2] = 11$, and $E[X_3^2] = 3.5$. Let $Y = X_1 - 2X_2 + 3X_3$ be a new random variable and find: (a) the mean value (10%), and (b) the variance of $Y$. (10%)

3. Let the random variables $X$ and $Y$ be independent and Gaussian, and let each have a mean of zero and a variance of $\sigma^2$.

$$f_X (x) = \frac{1}{\sqrt{2\pi \sigma^2}} \exp[-\frac{x^2}{2\sigma^2}]$$

If a new random variable $Z$ is defined by $Z = XY$ (a) Find the conditional probability density function of $Z$ given $Y$. (15%) (b) What is the probability density function of $Z$ (15%)?
1. (25%) 本題之計分標準以最後答案為準，不考慮計算過程。解答請寫在計算題部份，註明小題號，由(1)、(2)、⋯、依序列出。

Part I (15pts). A periodic signal \( x(t) \) has a period \( T = 2 \) and

\[
x(t) = \begin{cases} 
1, & |t| < \frac{1}{2} \\
0, & \frac{1}{2} \leq |t| < 1 .
\end{cases}
\]

Represent \( x(t) \) in complex Fourier series \( x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t} \),

where \( \omega_0 \) is the fundamental angular frequency. Find

(a) \( X[0] = \) __ (1) __
(b) \( X[0] - X[1] + X[2] - \cdots = \) __ (2) __
(c) \( \sum_{k=-\infty}^{\infty} X[k]^2 = \) __ (3) __

Part II (10pts). The Fourier transform of \( x(t) \) is defined as

\[
X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt .
\]

Suppose \( x(t) = \begin{cases} 
1, & |t| < \frac{1}{2} \\
0, & \text{otherwise}
\end{cases} \)

(a) Find \( X(j\pi) = \) __ (4) __
(b) Calculate the following convolution integral

\[
\int_{-\infty}^{\infty} \sin(\pi \tau) \sin(2\pi(t-\tau)) d\tau = \) __ (5) __

2. (40%) 本題含有兩個部分，每一部分分有 20 答案點。只接受答對的答案，若答錯則不扣分。

Part I is divided into two parts, each containing 20 points. Part II only accepts correct answers.

(a) If \( a > b \) and \( b > c \), then \( a > c \).
(b) Let \( A \in \mathbb{R}^{m \times n} \). Then matrix \( A \) is singular if and only if \( \det(A) = 0 \).
(c) Let \( A \in \mathbb{R}^{m \times n} \). Then all columns of matrix \( A \) are linearly dependent if and only if null space \( N(A) \neq \{0\} \).

根據數學和線性代數的知識，你知道(S1)是對的，S2和S3是錯的。因此你寫的答案如下：

answer for Part I:
wrong statement: correction (may be written in Chinese)

(S2) The statement is true only when \( A \) is a square matrix, i.e. when \( m = n \).
(S3) \( N(A) = \{0\} \) should be corrected as \( N(A) \neq \{0\} \).

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注意：
1. 更正的答案必須簡潔、清楚。因為每一題的錯誤處都不能整段重新寫。
2. 本題中，(S2)和(S3)的更正範圍在介紹中已寫明。3. 原答案中的任意部分，如果答對了，則可在後面更正。

此部份的分數計算方式為：
- 每一個正確的更正得 2 分。如果你所寫的更正不正確，則可得 0 分。
- 每一個錯誤的更正。如果更正不正確，則可得 0 分。如果更正正確，則可得 2 分。
-如果您答對了，則可得 20 分。如果答錯了，則可得 0 分。

因此，如果你的解答如上所述，則可得 10 分。但是，如果你的解答如下

answer for Part I:
wrong statement: correction (may be written in Chinese)

(S1) \( a > c \) should be corrected as \( a > c \).
(S3) \( N(A) = \{0\} \) should be corrected as \( N(A) \neq \{0\} \).

則你會得到 1 分。其餘情況可自行解決。
Part I (20%): Let $m$ and $n$ be any two positive integers and let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^n$ be arbitrary. We use $R(A)$ to denote the range or column space of $A$ and $N(A)$ to denote its null space. In the following (S1) to (S8) eight statements, at least four of them are wrong. Please choose arbitrary four statements that you think are WRONG and write the correction as simple and precise as possible in your answer.

(S1) Either $x \in \mathbb{R}^n$ to satisfy the linear system $Ax = b$ or $y \in \mathbb{R}^n \cap N(A^T)$ such that $\langle y, b \rangle = 0$.
(S2) Either $\langle b, Ax \rangle = 0$ for $\forall x \in \mathbb{R}^n$ or $\langle b, Ax \rangle > 0$ for every nonzero $x \in \mathbb{R}^n$.
(S3) Matrix $A$ has a right inverse, i.e. $\exists B \in \mathbb{R}^{n \times m}$ such that $AB = I_n$, if and only if $A$ is full row rank, i.e. all rows of $A$ are linearly independent. In that case, the linear system $Ax = b$ is always consistent, i.e. it is always solvable.
(S4) The linear system $A^T A x = A^T b$ is always consistent if and only if matrix $A$ is full row rank.
(S5) Matrix $A^T A$ is nonsingular and if and only if matrix $A$ is full row rank. In that case, the linear system $A^T Ax = b$ has at least one solution whenever it is consistent. When the linear system is inconsistent, however, it has a least squares solution described by the vector $x = (A^T A)^{-1} A^T b$, i.e. $\|Ax - b\|_2 \leq \|A^T x - b\|_2$ for $\forall x \in \mathbb{R}^n$.
(S6) Suppose $A$ is not a full rank matrix, i.e. assume $\text{rank}(A) = k < \min(m,n)$. Then $\exists Q \in \mathbb{R}^{m \times k}$ and $R \in \mathbb{R}^{k \times n}$ such that $A = QR$, where all columns of $Q$ are orthonormal and $R$ is a non-square upper triangular matrix.
(S7) From the QR factorization of a not full rank matrix $A$, i.e. $A = QR$ as mentioned in (S6), obviously we know $R(A) \subset R(Q)$. But the reverse inclusion $R(Q) \subset R(A)$ does not hold because of the existence of matrix $R$.
(S8) Let $L$ be the linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ defined by $A$, i.e. $L(x) := Ax$ for any $x \in \mathbb{R}^n$. Let $L^*$ be the linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ defined by $A^T$, i.e. $L^*(y) := A^T y$ for any $y \in \mathbb{R}^n$. Then $\forall y \in \mathbb{R}^n$ such that $L(y) = b$ if and only if $b \in \text{Ker}(L^*)$.

Part II (20%): Let $A$ be an $n \times n$ real matrix. Denote $S$ and $T$ as the symmetric and skew-symmetric parts of $A$, and let the eigenvalues of $AA^T$, $S$, and $T$ be $\alpha_i$, $\beta_i$, and $\gamma_i$ for $i = 1, \ldots, n$, respectively. Please answer following questions without giving any proof.

(a) (6%) What are $S$ and $T$ respectively?
(b) (2%) What is the value of $\text{trace}(ST - TS)$?
(c) (6%) What is the relationship between $\sum_{i=1}^n \alpha_i$, $\sum_{i=1}^n \beta_i$, and $\sum_{i=1}^n \gamma_i$?
(d) (6%) Without loss of generality, suppose $n = 3$ and let $\beta_1 = \beta_2$ and $\gamma_1 = \gamma_2$ be two pairs of distinct eigenvalues of $S$ and $T$, respectively. Let $u_i$ be a nonzero vector in the null space $N(S - \beta_j I)$ and, similarly, let $v_i$ be a nonzero vector in the null space $N(T - \gamma_j I)$ for $i = 1, 2, 3$. Then which are possible pairs of $u_i$'s and $v_i$'s?

(A) $u_1 = [1 \ 0 \ 0]^T$, $u_2 = [0 \ 1 \ -1]^T$ and $v_1 = [1 \ -1 \ 0]^T$, $v_2 = [0 \ 1 \ 0]^T$

(B) $u_1 = [1 \ 0 \ -1]^T$, $u_2 = [0 \ 1 \ 1]^T$ and $v_1 = [1 \ -1 \ 0]^T$, $v_2 = [0 \ 1 \ 0]^T$

(C) $u_1 = [1 \ 0 \ -1]^T$, $u_2 = [\sqrt{2} \ 1 \ \sqrt{2}]^T$ and $v_1 = [i \ 0 \ 0]^T$, $v_2 = [0 \ 0 \ 0]^T$

(D) $u_1 = [1 \ 0 \ 1]^T$, $u_2 = [-\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}]^T$ and $v_1 = [1 \ 1 \ 1]^T$, $v_2 = [0 \ -1 \ 0]^T$

(E) All of above statements are correct.
(F) None of above statements is correct.

3. (15%) Find the general solution of $x' = (x^2 + xy + y^2) dx$.

4. (20%) Find the Laplace transform of the following function: $t \ e^{t} \sinh 2t$
I. (20%) Identify two differences between the following terminology pairs.
(1) (5%) Distributed-Memory multicomputer vs Cluster Computer
(2) (5%) PowerPC architectures vs Itanium architectures
(3) (5%) Write-Through Cache vs Copy-Back Cache
(4) (5%) RAID-1 vs RAID-2

II. (20%) Instruction Set Architectures
(1) (10%) Assume an instruction set that uses a fixed 16-bit instruction length to provide
three types of instructions: zero-operand, one-operand, and two-operand. Operand
specifiers are 6 bits in length. There are K two-operand instructions and L
zero-operand instructions. What is the maximum number of one-operand instructions
that can be supported?
(2) (10%) Design a variable-length opcode to allow all of the following to be encoded
in a 36-bit instruction.
- 7 instructions with two 15-bit addresses and one 3-bit register number.
- 500 instructions with one 15-bit address and one 3-bit register number.
- 50 instructions with no addresses or registers.

III. (20%) Assume the exponent e is constrained to lie in the range 0 <= e <= X, with a bias of q, that the
base is b, and that the significand is p digits in length.
(1) (10%) What are the largest and the smallest positive values that can be written?
(2) (10%) What are the largest and the smallest positive values that can be written as normalized
floating-point numbers?

IV. (20%) Cache Memory Design
Consider a 32-bit microprocessor that has an on-chip 16 Kbytes four-way set associative cache.
Assume that the cache has a line (or block) size of four 32-bit words.
(1) (5%) Show the 32-bit physical address (Show how many Tag bits, Set bits, and Offset bits).
(2) (5%) Where in the cache (by indicating the set number) is the double word from memory location
ABCDE8F8 mapped?
(3) (10%) Draw a block diagram of this cache showing its organization and how the different address
fields are used to determine a cache hit or miss.

V. (20%) A two-level memory system has eight (page 0 to page 7) virtual pages on a disk to be mapped
into four page frames (PFs) in the main memory. Assume each page has 4 words.
The i-th page in disk consists of word address from 4i to 4i+3, where i = 0 to 7.
A certain program trace generated the following word address.

0, 1, 2, 5, 6, 7, 8, 9, 3, 4, 9, 10, 4, 5, 10, 11, 6, 7, 13, 12, 16, 17, 14.
(1) (10%) Show the successive virtual pages residing in the four page frames using FIFO (First-In-First-Out). Assuming that the main memory PFS are empty initially.
Compute the Hit Ratio.
(2) (10%) Repeat the above using LRU (Least Recently Used). Compute the Hit Ratio.
1. (35%) 本題之計分僅以最後答案為準，不考慮計算過程。答案請寫在計算題部份，註明小題號，由(1)、(2)、... 依序列出。
   Part I (15 pts). A periodic signal \( x(t) \) has a period \( T = 2 \) and
   \[
   x(t) = \begin{cases}
   1, & |t| < \frac{1}{2} \\
   0, & \frac{1}{2} \leq |t| < 1
   \end{cases}
   
   Represent \( x(t) \) in complex Fourier series \( x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{j2\pi k t} \),
   
   where \( \omega_0 \) is the fundamental angular frequency. Find
   
   (a) \( X[0] = \) (1). (b) \( X[0] - X[1] + X[2] - \cdots = \) (2). (c) \( \sum_{k=-\infty}^{\infty} X[k] = \) (3).

   Part II (10 pts). The Fourier transform of \( x(t) \) is defined as
   \[
   X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.
   
   Suppose \( x(t) = \begin{cases}
   1, & |t| < \frac{1}{2} \\
   0, & \text{otherwise}
   \end{cases}
   
   \]
   Find \( X(j\pi) = \) (4). (b) Calculate the following convolution integral
   \[
   \int_{-\infty}^{\infty} \frac{\sin(\pi r)\sin(2\pi(1-r))}{\pi r(1-r)} dr = \) (5).

   Part III (10 pts). Let field \( \vec{G}(x,y,z) = \vec{a}_i(x-xz) + \vec{a}_j(y^2-zx) + \vec{a}_k(x^2-xy) \).
   
   (a) Find the line integral \( \int_{C_1} \vec{G}(x,y,z) \cdot d\vec{l} = \) (6), where \( C_1 \) is a segment of the curve
   \( y = x^2, x = x \) from \((0,0,0)\) to \((1,1,1)\).
   (b) Compute the sum of line integrals \( \int_{C_2} \vec{G}(x,y,z) \cdot d\vec{l} + \int_{C_3} \vec{G}(x,y,z) \cdot d\vec{l} \) where \( C_2 \) is the straight line from \((1,1,1)\) to \((0,0,1)\) and \( C_3 \) is along the \( z \)-axis from \((0,0,1)\) to \((0,0,0)\).
   
   Answer: (7).

2. (30%) This problem contains two parts with 15 points in each part. ONLY ANSWERS WITHOUT PROOF are required in both parts.

   Part I is to select five true statements. For each false statement, give the correct answer and explain why the original statement is false. Each correct answer is worth 5 points.
   
   (S1) If \( a > b \) and \( b > c \), then \( a > c \).
   (S2) Let \( A \in \mathbb{R}^{m \times n} \). Then matrix \( A \) is singular if and only if \( \det(A) = 0 \).
   (S3) Let \( A \in \mathbb{R}^{n \times n} \). Then all columns of matrix \( A \) are linearly dependent if and only if null space \( N(A) \neq \{0\} \).

   Answer:
   
   Part I:
   
   **Answer for Part I:**
   
   **Wrong statement:** correction (may be written in Chinese)
   
   (S2) The statement is true only when \( A \) is a square matrix, i.e. when \( m = n \).
   
   (S3) \( N(A) = \{0\} \) should be corrected as \( N(A) \neq \{0\} \).

   **Note:**
   
   1. 更正的敘述必須簡潔、精確；因為每個敘述中錯誤處的更正都僅需寫一行即可完成。
   2. 當然，您也必須指出(S3)是錯誤的，並且更正為"only if part"敘述中的"linearly dependent"改成"linearly independent"。此更正亦適用於(S2)及(S3)。更正為"only if part"敘述的直接和反敘述是錯誤的，且更正為"if part"或"if part"中的敘述即在正確時，更正為"if part"中的敘述，不遵守此一規則。更正的答案一律視為正確的答案。

   此部分的分數計算方式為：
   - 每一個正確的選擇為 2 分，如果你提供的更正是正確的，則可再得 3 分 (因此，每個正確的選擇及正確的更正共可得 5 分，而如此答對四個正確的選擇及更正，便可得到 20 分)。如果選擇正確，但是所提供的
Part I (15%): Let $m$ and $n$ be any two positive integers and let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^n$ be arbitrary. We use $R(A)$ to denote the range or column space of $A$ and $N(A)$ to denote its null space. In the following (S1) to (S6) six statements, at least three of them are wrong. Please choose arbitrary three statements that you think are WRONG and write the correction as simple and precise as possible in your answer.

(S1) Either exist $x \in \mathbb{R}^n$ to satisfy the linear system $Ax = b$ or exist $y \in \mathbb{R}^n \cap N(A^T)$ such that $(y, b) \neq 0$.

(S2) Either $(b, Ax) = 0$ for $\forall x \in \mathbb{R}^n$ or $(b, Ax) > 0$ for each nonzero $x \in \mathbb{R}^n$.

(S3) Matrix $A$ has a right inverse, i.e. $\exists B \in \mathbb{R}^{n \times m}$ such that $AB = I_n$, if and only if $A$ is full row rank, i.e. all rows of $A$ are linearly independent. In that case, the linear system $Ax = b$ is always consistent, i.e. it is always solvable.

(S4) The linear system $A^TAx = A^Tb$ is always consistent if and only if matrix $A$ is full row rank.

(S5) Matrix $A^TA$ is nonsingular if and only if matrix $A$ is full row rank. In that case, the linear system $Ax = b$ has at least one solution whenever it is consistent. When the linear system is inconsistent, however, it has a least squares solution described by the vector $\hat{x} := (A^TA)^{-1}A^Tb$, i.e. $\|A\hat{x} - b\|_2 \leq \|Ax - b\|_2$ for $\forall x \in \mathbb{R}^n$.

(S6) Let $L$ be the linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ defined by $A$, i.e. $L(x) := Ax$ for any $x \in \mathbb{R}^n$. Let $L^*$ be the linear transformation from $\mathbb{R}^m$ to $\mathbb{R}^n$ defined by $A^T$, i.e. $L^*(y) := A^Ty$ for any $y \in \mathbb{R}^m$. Then $\exists v \in \mathbb{R}^n$ such that $L(v) = b$ if and only if $b \in Ker(L^*)$.

Part II (15%): Let $A$ be an $n \times n$ real matrix. Denote $S$ and $T$ as the symmetric and skew-symmetric parts of $A$, and let the eigenvalues of $AA^T$, $S$, and $T$ be $\alpha_i$, $\beta_i$, and $\gamma_i$ for $i = 1, \ldots, n$, respectively. Please answer following questions without giving any proof.

(a) (6%) What are $S$ and $T$ respectively?

(b) (2%) What is the value of $\text{trace}(ST - TS)$?

(c) (7%) What is the relationship between $\sum_i \alpha_i$, $\sum_i \beta_i$, and $\sum_i \gamma_i$?

3. (15%) Evaluate $\int_{C} \frac{z + 8}{z^2 + z - 2} \, dz$ by using the residue theorem, where $C : |z| = 3$ and $z = x + iy$

4. (20%) Find the general solution of $(x^2 + xy + y^2) \, dx - x^2 \, dy = 0$. 
1. For the circuit shown in Fig 1, (a) break the feedback loop at X and find the loop gain $\beta A$, then (b) find the frequency of oscillation $f_o$ and (c) the minimum required value of $R_f$ for oscillations to start in the circuit.  

2. In Figure 2 an inverter fabricated in a 0.12μm CMOS technology uses the minimum possible channel lengths (i.e. $L_n = L_p = 120$nm). (a) If $W_n = 180$nm, find the value of $W_p$ that would result in $Q_n$ and $Q_p$ being matched. (b) For this technology, $k'_n = 160\mu A/V^2$ and $k'_p = 54\mu A/V^2$ with the supply voltage $V_{DD} = 1.5$ V and the threshold voltage $V_{th} = 0.5$V. Calculate the value of the output resistance of the inverter when $v_o = V_{OL}$ ("Low" level at the output).  

3. Consider the source-follower circuit shown in Figure 3. The most negative output signal voltage occurs when the transistor just cuts off. Show that (a) this output voltage $v_o(min)$ and (b) the corresponding input voltage $v_i(min)$ with respect to $I_{DO}$, $g_m$, $R_S$ and $R_L$.  

Figure 1.

Figure 2.

Figure 3.
4. Figure 4 shows an op amp connected in the inverting configuration. The op amp has an open-loop gain $\mu = 10^4$, a differential input resistance $R_{id} = 100\, \text{k}\Omega$, and an output resistance $r_o = 1\, \text{k}\Omega$. Use the feedback method to find (a) the voltage gain $V_o/V_i$, (b) the input resistance $R_{in}$, and (c) the output resistance $R_{out}$.

5. (a) Using a simple ($r_s$ and $g_m$) model for each of the two transistors $Q_{18}$ and $Q_{19}$ in Figure 5.a, find the small-signal resistance between $A$ and $A'$. Where $I_{C18} = 165\, \mu\text{A}$ and $I_{C19} = 16\, \mu\text{A}$. 10%  
(b) Figure 5.b shows the circuit for determine the 741 op-amp output resistance when $v_o$ is positive and $Q_{14}$ is conducting most of the current. Using the resistance of the $Q_{18}$ and $Q_{19}$ network calculated in 5.(a) and neglecting the large output resistance of $Q_{13}$, find $R_o$ when $Q_{14}$ is sourcing an output current of 5mA. 10%
1. Answer the following true-and-false problems. Correct each statement if your answer to that statement is "false". No credit will be given if you reply to the question only with the answer "false", or your corrected answer is not correct.

(a)(4%) The initial-value theorem is applicable to linear time-varying systems.

(b)(4%) The final-value theorem is applicable to any linear time-invariant systems.

(c)(4%) Consider the following MIMO feedback control system, where \( R(s) \in \mathbb{R}^{p \times 1} \), \( Y(s) \in \mathbb{R}^{p \times 1} \). We can obtain the closed-loop transfer function matrix as

\[
M(s) = \left[ I + H(s)G(s) \right]^{-1}G(s).
\]

(d)(4%) If the transfer function of a control system has pole-zero cancellation, then this system is not controllable, but is still observable.

(e)(4%) When we apply parallel decomposition technique to a transfer function (no pole-zero cancellation) of a control system, we can always obtain a set of state equations with diagonal canonical form.

(f)(4%) Adding a zero to a second-order prototype closed-loop transfer function will generally decreases both the rise time and the maximum overshoot of the step response.

(g)(4%) The asymptotes of the root loci \( 1 + kG(s)H(s) \) refer to the angles of the root loci when \( k = \pm \infty \).

(h)(4%) Bode plot can be used for stability analysis both for minimum and nonminimum-phase transfer function.

(i)(4%) Assigning all the eigenvalues to the left-half of \( s \)-plane still can not guarantee the stability of a linear time-varying system.

(j)(4%) Nichols chart can be used to find bandwidth and the resonant peak value of an open-loop system.

2.(10%) Suppose that a designer wants to analyze a control system whose signal-flow graph (SFG) is shown below.

Now he wants to use the Mason rule \( M = y_{out}/y_{in} = \sum_{k=1}^{N} M_k \Delta_k / \Delta \) to compute the transfer function \( y_{out}/y_{in} \), where \( N \) is the total number of forward paths between \( y_{in} \) and \( y_{out} \), \( M_k \) is the gain of the \( k \)-th forward paths between \( y_{in} \) and \( y_{out} \), and

\[
\begin{align*}
\Delta &= 1 - \text{(sum of the gains of all individual loops)} \\
&+ \text{(sum of products of gains of all possible combinations of two noncoching loops)} - \cdots \\
\Delta_k &= \text{the} \Delta \text{for that part of the SFG that is noncoching with the} \ k \text{th forward path}
\end{align*}
\]
The computation results of this designer are shown as follows:

\[ N = 2, \]
\[ M_1 = a_{23}a_{34}, \quad M_2 = a_{24} \]
\[ \Delta = 1 - (a_{32}a_{43} + a_{24}a_{43}a_{32} + a_{44}) + a_{23}a_{32}a_{44} \]
\[ \Delta_1 = 1, \quad \Delta_2 = 1 \]

Are the computations correct or not? Point out which step you think that is not correct, explain your reason, and re-compute the transfer function \( y_4/y_2 \).

3. (10%) For the feedback system of Fig. 1, please design a second-order \( G(s) \) such that the overall feedback system is stable and the steady-state tracking error \( e \) is zero when applied with \( r(t) = \sin(2t) \).

4. Consider the feedback system of Fig. 1 with the Nyquist diagram of \( G(j\omega) \) sketched in Fig. 2, where point A is \([-0.2+j0] (\omega = 800 \text{ rad/sec}) \), point B is \([-\sqrt{3} + j]/4 (\omega = 600 \text{ rad/sec}) \), point C is \([-1 + j]/\sqrt{2} (\omega = 200 \text{ rad/sec}) \). Assume \( G(s) \) has no poles in the right half s-plane.

(1) (5%) Please determine the stability of the feedback system from its Nyquist diagram. 
(2) (10%) Find its gain margin and phase margin.
(3) (5%) Suppose input \( r \) is a unit step function, please find its steady-state regulation error \( e(t) \).
(4) (5%) Suppose \( r(t) = \cos(600t) \), please find its steady-state output response \( y(t) \).

5. Consider the system of Fig. 1, where \( G \) is described by

\[ \dot{x} = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} e, \quad y = [k \quad 0] \]

(1) (5%) Find the transfer function \( G(s) \).
(2) (5%) Determine the range of \( k \) for a stable closed-loop system.
(3) (5%) Determine the range of \( k \) so that no overshoot in output \( y(t) \) is found with a step input \( r \).
**Physical constants:**

\[ K = 1.38 \times 10^{-23} \text{ J/K} \quad e = 1.60 \times 10^{-19} \text{ C} \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]

\[ kT(300 \text{ K}) = 0.0259 \text{ V} \quad h = 6.625 \times 10^{-34} \text{ J-s} \quad \text{Eg; Si} = 1.12 \text{ eV} (300 \text{ K}) \]

\[ \varepsilon_{\text{Si}} = 11.7 \varepsilon_0 \quad (\text{Si; } m_n^* = 1.08 \text{ mo} ; m_p^* = 0.56 \text{ mo}) \]

\[ \text{GaAs; } N_c = 4.45 \times 10^{17} \text{ cm}^{-3} ; \quad N_v = 7.72 \times 10^{18} \text{ cm}^{-3} \]

1. Consider a GaAs MESFET with a gold Schottky barrier with a barrier height of 0.8 V. The n-channel doping is \(10^{17} \text{ cm}^{-3}\) and the channel thickness is 0.25 \(\mu\text{m}\). Calculate the 300 K threshold voltage for the MESFET. (15%)

2. To calculate the built-in potential barrier, the space charge width and electric field in a p-n junction. Consider a silicon p-n junction at \(T=300 \text{ K}\) with doping densities; \(N_a=1 \times 10^{16} \text{ cm}^{-3}\) and \(N_d = 1 \times 10^{15} \text{ cm}^{-3}\). (20%)

3. Consider a GaAs p-n diode with the following parameters at 300 K:

   - Electron diffusion coefficient: 30 \(\text{cm}^2/\text{V-s}\)
   - Hole diffusion coefficient: 15 \(\text{cm}^2/\text{V-s}\)
   - p-side doping: \(5 \times 10^{16} \text{ cm}^{-3}\)
   - n-side doping: \(5 \times 10^{17} \text{ cm}^{-3}\)
   - Electron mobility carrier lifetime: \(\tau_n = 10^{-8} \text{ s}\)
   - Hole mobility carrier lifetime: \(\tau_p = 10^{-7} \text{ s}\)

   Calculate the injection efficiency of the LED assuming no recombination due to traps. (15%)

4. Calculate the probability that a state in the valence band is occupied by a hole; assume the Fermi energy is 0.25 eV below the conduction band. and calculate the thermal equilibrium hole concentration in Silicon at 300 K. (20%)

5. Derive the Einstein relation. (10%)

6. To determine the diffusion coefficient given the carrier mobility. Assume that the mobility of a particular carrier is \(1000 \text{ cm}^2/\text{V-s}\) at \(T = 300 \text{ K}\). (10%)

7. Compare the properties of the p-n diode and the Schottky diode. (10%)