1) **Quadratic equations** [10 marks]

Consider the quadratic equation \( ax^2 + bx + c = 0, \quad a \neq 0. \)

(a) Find the quadratic formula for \( x. \)

(b) Use the *discriminant* of the quadratic equation, state when the equation will have two real and distinct roots, two real and equal roots, or two distinct imaginary numbers.

2) **Quadratic equation** [5 marks]

For a rectangular with length \((6-x)\) and width \(x\), express its area as quadratic function of \(x.\)

For what value of \(x\) will the area be a maximum?

3) **Operation with complex numbers** [5 marks]

Given \( f(z) = z^2 + 6z + \frac{1}{z}, \) where \( z \) is a complex number. Find \( f(3i). \)

4) **Inverse variation** [10 marks]

If \( y \) varies inversely as the cube root of \( z, \) and \( y = 2 \) when \( z = 2, \) find \( y \) when \( z = 8. \)

5) **Radical equation** [5 marks]

Solve radical equation \( \sqrt{x+6} - \sqrt{2x+5} = -1. \)

6) **Radical equation** [10 marks]

Solve \( \frac{1}{x^2} - \frac{8}{x^2} + 7 = 0. \)

7) **Logarithmic equation** [5 marks]

Solve \( \log_e (2x + 4) = 3. \)

8) **Limits** [10 marks]

(a) \( \lim_{x \to 0} \frac{\sin(x)}{x} \)

(b) \( \lim_{x \to 2} \frac{\ln(2x - 3)}{x^2 - 4} \)
9) 【Derivative by chain rule】【10 marks】

Apply the chain rule to find \( \frac{du}{dt} \) given \( u = x^2 + 2xy + y^2 \) where \( x = t \cos t \) and \( y = t \sin t \).

10) 【Integration by substitution】【10 marks】

Find the indefinite integral of \( \int (x^2 \sqrt{x^3 + 4}) \, dx \).

11) 【Integration of trigonometric functions】【5 marks】

Find the indefinite integral of \( \int \sin^3 x \cos x \, dx \).

12) 【Application of differential calculus】【15 marks】

Sand is falling into a conical pile at the rate of 5 \( \text{ft}^3/\text{min} \). If the height of the pile is always twice the radius of the base, how fast is the height increasing when the pile is 3 \( \text{ft} \) high?

【Hint: The volume of the cone (pile) at any time is expressed as \( V = \frac{1}{3} \pi r^2 h \), where \( r \) is the radius and \( h \) is the height】
第一部份：定義/數學公式/簡要說明題【50分】
1. 【Definition/Equation/Brief Answer : 5% each】
   (1). How can we measure the shear stress in laboratory condition?
   (2). What is a Venturi tube?
   (3). Continuity equation in a two-dimensional incompressible flow.
   (4). Bernoulli equation in a two-dimensional incompressible flow.
   (5). Froude number.
   (6). Absolute pressure.
   (7). Path line of a fluid particle.
   (8). Control volume.
   (9). State the three laws of similarity in modeling a prototype condition.
   (10). Main difference between Eulerian and Lagrangian description of fluid motion.

第二部份 ：計算題 【50分】
2. 【Streamline : 10%】
   Given the streamline equation for a fluid particle in a two-dimensional flow field as
   \[
   \frac{dy}{dx} = \frac{\nu}{u}.
   \]
   Find the streamline equation when a particle passes the point (1, 1), if the velocity
   components are given by \(u = x\) and \(v = y\).

3. 【Hydrostatic force : 10%】
   A water tank as shown is completely filled with water with density 1000 kg/m³. Calculate the water
   force on the slanted side wall ABCD of the tank and the location of this force.

4. 【Energy equation : 10%】
   Determine the power required for a pump to deliver 0.05 m³/s of water at atmospheric
   pressure through a 100-mm pipe to a building 100 m above sea level. Assume the density of
   water as 1000 kg/m³, and neglect heat transfer and all internal energy losses when the flow
   through the pipe.
   【Hint: The energy equation applicable may be written as
   \[
   -\frac{dW}{dt} = \rho AV \left[ \Delta \left( \frac{P}{\rho} + \frac{V^2}{2} + gz \right) \right],
   \]
   where \(W\) is the work done on the fluid and \(\Delta\) denotes the
   difference of total energy \(\left( \frac{P}{\rho} + \frac{V^2}{2} + gz \right)\) between the discharge and the entrance points.】
5. **[Dimensional analysis and similitude: 10%]**

Air flows with an average velocity of 10 m/s through a circular pipe having diameter of 250 mm, under the condition at 1 atmospheric pressure and 20°C.

(1). What must be the average velocity in a model of this flow to be reproduced in a water pipe of 60-mm in diameter, if the flow is dynamically similar to the prototype?

(2). Find the pressure drop in the prototype, if the pressure drop in the model is 200 kPa.

**[Hint 1]:** In (1), equate Reynolds number for the prototype and model, and in (2) let \( \Delta p = \rho V^2 \).

**[Hint 2]:** Additional data: kinematic viscosities \( \nu_{air} = 1.51 \times 10^{-5} \text{ m}^2/\text{s}, \nu_{water} = 1 \times 10^{-6} \text{ m}^2/\text{s} \); densities \( \rho_{air} = 1.204 \text{ kg/m}^3, \rho_{water} = 998.3 \text{ kg/m}^3 \).

6. **[Velocity distribution in a typhoon: 10%]**

The eye of a typhoon (tropical cyclone) has a radius \( R \) of 20 m and the maximum wind velocity at the edge of the eye is 50 m/s. Find the variation of tangential velocity in the flow field of the typhoon, for the radial distance \( r \leq R \) (assume rigid body rotation, i.e., forced vortex) and \( r \geq R \) (free vortex), respectively.
國立中山大學海洋物理研究所碩士班入學考試 海洋物理學試題 （總分 100 分）

1. 潮汐（30%）
   (a) 何謂大潮？何謂小潮？何謂潮流週期？何謂引潮力？（10%）
   (b) 請甲簡著掘淺海（如澎湖地區）海水之潮汐運動時，是否需考慮引潮力？（10%）
   (c) 以下為澎湖馬公地區之潮位紀錄，請描述其特性（溼差、週期等）。（10%）

2. 波浪（20%）
   (a) 波浪由深海傳至淺海，請問波浪波之適用範圍為何？其相位速度為何？能量傳波之速度為何？（10%）
   (b) 在海邊看到的波浪幾乎都是與海岸垂直，請問這是水波的何種效應？在受障礙物遮蔽的區域也有波動，請問這是水波的何種效應？（10%）

3. 假設香港外海發生地震後產生海嘯，若震央距高雄外海測站500公里，當地水深200公尺，高雄外海測站水深20公尺，請估計發生地震多少時間後海嘯會抵達高雄外海測站。（10%）

4. 解釋名詞（25%）
   (a) El Nino
   (b) 科氏力
   (c) 1節 = ? m/sec
   (d) Ekman spiral
   (e) Internal Wave

5. 試繪出海水溫度之典型剖面圖與聲速分布圖，以上兩圖有何相關？何謂 Sound Channel？（15%）