國立中山大學經濟研究所碩士班招生考試試題

科 目：總體經濟學【經濟所碩士班】

國立中山大學經濟學研究所碩士班招生考試「總體經濟學」試題

符號說明：

- $Y$ = 實質所得
- $C$ = 實質消費
- $I$ = 實質投資
- $R$ = 利率
- $P$ = 物價
- $M$ = 貨幣數量
- $L$ = 實質貨幣需求
- $t$ = 時間
- $X'$ = 变數X之時間導數
- $X_t$ = 第t期之X值
- $X = M, Y, P, C, I, ...$
- $X_{t+1}^t$ = 在第t+1期時對X_t所形成之預測值
- $e_t$ = 第t期總合供給之外生變數干擾
- $\epsilon_t \sim N(0, \sigma^2_t)$, $\sigma^2_t = \text{constant}$

請回答下列問題：

一、(30%)

證分析以下總體經濟模型

**IS**: $Y = C(Y') + I(R)$, $0 < C' < 1, \quad I' < 0$

**LM**: $\frac{M}{P} = L(Y, R), \quad L_Y = \frac{\partial L}{\partial Y} > 0, \quad L_R = \frac{\partial L}{\partial R} < 0$

以導出總合需求函數 $Y = Y^d(M, P)$，其中

$$dY = \frac{1}{\alpha} \frac{dM}{P} - \frac{1}{\alpha} \frac{M}{P} dP,$$

而

$$\frac{1}{\alpha} = \frac{I'}{(1 - C')L_R + L_Y I'}.$$

二、(20%)

考慮如下之總合供給函數

**AS**: $Y_t - Y_{t-1} = \beta(R_t - P_{t-1}^d) + \epsilon, \quad \beta > 0$

**AD**: $P_t - P_{t-1} = -\alpha(Y_t - Y_{t-1}) + M_t - M_{t-1}, \quad \alpha > 0$

假設 $P_{t+1}^d$ 為外生因素所決定之固定值，推算貨幣政策乘數 $\frac{dY_t}{dM_t}$ 與 $\frac{dP_t}{dM_t}$。

三、(30%)

請考慮如上述第二題之模型，但令 $P_{t+1}^d$ 為依照「理性預期假設 (Rational Expectations Hypothesis)」所形成之價格預測值。請推算此預測價格 $P_{t+1}^d$ 之形成方程式。

四、(20%)

續上述第三題，假設貨幣供給函數為 $M_t = \tilde{M}_t + \tilde{M}_t$, 其中 $\tilde{M}_t$ 與 $\tilde{M}_t$ 分別代表在第t-1期時為「可正確預測」與「不可正確預測」之第t期貨幣數量的成分，亦即:

$$\tilde{M}_t = \hat{M}_t, \quad \hat{M}_t = \mu_t, \quad \mu_t \sim N(0, \sigma^2_t), \quad \sigma^2_t = \text{constant}.$$

在這些貨幣供給條件下根據「理性預期假設」差算：

1. 「事先可正確預測之貨幣政策」乘數 $\frac{dY_t}{d\tilde{M}_t}$ 與 $\frac{dP_t}{d\tilde{M}_t}$
2. 「事先不可正確預測之貨幣政策」乘數 $\frac{dY_t}{d\hat{M}_t}$ 與 $\frac{dP_t}{d\hat{M}_t}$
1. Explain the following terms:
   (a) Nash equilibrium; (5%)
   (b) Transaction costs and Coase Theorem; (5%)
   (c) Pareto efficiency; (5%)
   (d) Arrow's Impossibility Theorem. (5%)

2. In a duopoly, two firms involve in Cournot competition. The cost function of each firm is given by: \( c_i(q_i) = c q_i, i = 1,2 \). The market demand function is:

   \[ p = a - (q_1 + q_2) \]

   (a) What is the Cournot-Nash equilibrium? (5%)
   (b) What will the outcome be if the two firms form a cartel? (5%)
   (c) What will the outcome be if firm 1 acts as a Stackelberg leader? (5%)
   (d) What will the outcome be if firm 1 acts as a price leader? (5%)
   (e) What will the outcome be if the two firms are involved in Bertrand competition? (5%)

3. There are \( n \) agents with identical utility functions, \( u_i(G, x_i) = G^a x_i^{1-a} \).

   Suppose that a total amount of wealth \( w \) is about to be equally divided among \( k \leq n \) of the agents.

   (a) How much of the public good is provided? (10%)
   (b) How does the amount of the public good change as \( k \) increases? (5%)

4. There are two players, a seller and a buyer, and two dates. At date 1, the seller chooses his investment level \( I \geq 0 \) at cost \( I \). At date 2, the seller may sell one unit if a good and the seller has cost \( c(I) \) of supplying it, where \( c'(0) = -\infty, c' < 0, c'' > 0 \), and \( c(0) \) is less than the buyer's valuation. There is no discounting, so the socially optimal level of investment, \( I^* \), is given by \( 1 + c'(I^*) = 0 \).

   (a) Suppose that at date 2 the buyer observes the investment \( I \) and makes a take-it-or-leave-it offer to the seller. What is this offer? (5%)
   (b) What is the perfect Nash equilibrium of the game? (10%)
   (c) Can you think of a contractual way of avoiding the inefficient outcome of (a)? (Assume that contract cannot be written on the level of \( I \).) (5%)
5. Two consumers each with an expected utility function of $\ln w$ and $\sqrt{w}$ respectively are offered a gamble. Each consumer initially has wealth $w$. If one bets $Sx$, he will have $w+x$ with a probability $\pi$, and $w-x$ with a probability $(1-\pi)$. For each consumer, solve for the optimal $x$ as a function of $\pi$. (10%) 

6. Suppose that a competitive industry faces a randomly fluctuating price for its output. For simplicity we imagine that the price of output will be $p_1$ with probability $q$ and $p_2$ with probability $(1-q)$. It has been suggested that it may be desirable to stabilize the price of output at the average price $\tilde{p} = qp_1 + (1-q)p_2$. True or false? Explain why? (10%)
Answer the following four questions, equally weighted

1. (25%) Let the $3 \times 1$ random vector $\mathbf{x}_i = (X_1, X_2, X_3)^T$ follow a multivariate normal distribution,

$$
\begin{bmatrix}
X_1 \\
X_2 \\
X_3
\end{bmatrix}
\sim \mathcal{N}(\mu, \Sigma),
$$

where

$$
\mu = \begin{bmatrix}
170 \\
68 \\
40
\end{bmatrix}
\quad \text{and} \quad
\Sigma = \begin{bmatrix}
400 & 64 & 128 \\
64 & 16 & 0 \\
128 & 0 & 256
\end{bmatrix}.
$$

Find
(a) The conditional distribution of $X_1$ given $X_2 = 72$, i.e. $f(X_1|X_2 = 72)$ and
(b) The conditional distribution of $X_1$ given $X_2 = 72$ and $X_3 = 24$, i.e. $f(X_1|X_2 = 72, X_3 = 24)$.

2. (25%) Let $X_1, X_2, X_3$ be independent with $X_i$ having density $f(x_i) = \exp(-x_i), x_i > 0; \forall i = 1, 2, 3$. Let $U_1 = X_1 + X_2 + X_3, U_2 = X_2/U_1$, and $U_3 = X_3/U_1$. Find the joint density of $U_1, U_2, U_3$.

3. (25%) (This is a question of Bayesian Statistics.) Let $X_1, \cdots, X_N$ be a sample from a normal distribution with mean $\Theta$ and variance one, and let $\Theta \sim \mathcal{N}(\alpha, \beta^2)$. Find the posterior distribution of $\Theta$ given $X_1, \cdots, X_N$.

4. (25%) Suppose that $X_1, \cdots, X_N$ form a random sample from a uniform distribution on the interval $(\theta_1, \theta_2)$, where both $\theta_1$ and $\theta_2$ are unknown and $-\infty < \theta_1 < \theta_2 < \infty$. Find the maximum likelihood estimators (MLE's) of $\theta_1$ and $\theta_2$. 

1