From question 1 to 5, each question scores 10 points.

1. Suppose that you are the consultant to an agricultural cooperative that is deciding whether members should cut their production of oranges in half next year. The cooperative wants your advice as to whether this action will increase members' revenues. Knowing that oranges ($O$) and watermelons ($W$) both compete for agricultural land, you estimate the demand for oranges to be,

$$O = 3.5 - P_o + 0.25P_w + 0.5I,$$

where $P_o$ is the price of orange, $P_w$ the price of watermelon, and $I$ income.

Should you support or oppose the plan? Is there any additional information that would help you to provide a definite answer?

2. Suppose that Mary's utility function is given by $u(I) = \sqrt{10I}$, where $I$ represents her income.

a) Suppose that Mary's current income is 40 and can earn that income next year with certainty. She is offered a chance to take a new job with 0.6 probability of earning 44 and 0.4 probability of earning 33. Should she take the new job?

b) Would Mary be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance?

3. A firm has a production process in which the inputs to production are perfectly substitutable in the long run. Can you tell whether the marginal rate of technical substitution is high or low, or is further information necessary? Discuss.

4. A sales tax of 10 percent is placed on half the firms (the polluters) in a competitive industry. The revenue is paid to the remaining firms (the nonpolluters) as a 10 percent subsidy on the value of output sold.

a) Assuming that all firms have identical constant long-run average costs before the sales tax-subsidy policy, what do you expect to happen (in both the short run and the long run), to the price of the product, the output of firms, and industry output?

b) Can such a policy always be achieved with a balanced budget in which tax revenues are equal to subsidy payments? Why or why not? Explain.
5. Two firms compete by choosing price. Their demand functions are

\[ \begin{align*}
Q_1 &= 20 - p_1 + p_2, \\
Q_2 &= 20 + p_1 - p_2,
\end{align*} \]

where \( p_1 \) and \( p_2 \) are the prices charged by each firm, respectively, and \( Q_1 \) and \( Q_2 \) are the resulting demands. Suppose you are one of these firms and that there are three ways you could play the game: (i) Both firms set price at the same time; (ii) You set the price first; or (iii) Your competitor sets price first. If you could choose among these options, which would you prefer? In the case you choose, what is the price you charge? Explain why.

6. 考慮 Cournot duopoly competition (雙寡)。假設需求函數為 \( D(p) = a - p \)，其中 \( a > 0 \) 並且 \( p \) 是價格。讓 \( c \) 代表邊際成本而且 \( a > c > 0 \)。

(6-1) 在 Cournot competition 下，兩家廠商分別之最大產量為何？（10 pts）

(6-2) 如果只有一家廠商 (獨佔)，找出最大生產量。（5 pts）

(6-3) 如果兩廠商一開始各自均生產獨佔產量的一半，而現在其中一家廠商決定改變策略，新策略將以極大化自己本身利益為前提，試問新產量為何？（假設另一家廠商維持產量不變）（10 pts）

7. 考慮一個兩個人、兩商品的經濟體系。第一個人的原賦為 \( e^1 = (0, 1) \)，而第二

個人的原賦為 \( e^2 = (1, 0) \)。設 \( x^k_i \) 代表第 \( i \) 個人擁有第 \( k \) 個商品的數量。

(7-1) 假設第 \( i \) 個人的效用函數為 \( u_i(x) = x_i x^k_i \)。找出唯一的 Walrasian equilibrium

分配。（10 pts）

(7-2) 假設其中一人的效用函數變為 \( \overline{u_i}(x) = x_i^k - \frac{1}{1 + x_i^k} \)，找出新的 Walrasian

equilibrium 分配。（10 pts）

(7-3) 請問改變效用函數對此人的影響為何？簡述之。（5 pts）
Answer the following five questions, equally weighted

1. (20%)
Let $X$ be uniformly distributed on the interval $(0,1)$, and let the conditional distribution of $Y$ given $X = x$ be uniformly distributed on the interval $(-x, x)$.
(a) Find the joint density of $(X, Y)$.
(b) Find the conditional density of $X$ given $Y = y$.

2. (20%)
Let $f(x, y, z) = 48xyz$, $0 < x < y < z < 1$. Find the following conditional variances:
(a) $\text{Var}(Y | Z = z)$ and
(b) $\text{Var}(Y | (X, Z) = (x, z))$.

3. (20%)
Let $X_1, \ldots, X_n$ be independent and $X_i \sim N(\theta, \sigma^2)$, where $\theta$ and $\sigma^2 > 0$ are unknown parameters. Find the maximum likelihood estimators (MLE) of $\theta$ and $\sigma^2$.

4. (20%)
In a sample of size 3 we observe 4, 6 and 8. Can we reject the hypothesis that $\mu = 3$ against $\mu \neq 3$ with these data? Use $\alpha = 0.05$.

5. (20%)
Suppose that $X_n \xrightarrow{p} 3$ (read as $X_n$ converges in probability to 3) and $Y_n \xrightarrow{p} 6$.
To what do the following converge in probability? State the reason.
(a) $\frac{n(X_n + Y_n)}{n + 1}$, and
(b) $\frac{n^2(X_n + Y_n)}{n^2 + n - 3}$.
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**TABLE CONTENTS:**
- Statistical Tables
- Appendix G: Standard Error Tables

**NOTES:**
- This table contains statistical data related to various fields of study.
- The tables are organized in a structured format, making it easy to read and analyze.
- The data is presented in a clear and concise manner, ensuring easy comprehension.

**LIMITATIONS:**
- The data may not cover all aspects of the topic discussed.
- More detailed information can be found in Appendix G: Standard Error Tables.
1. From the following table, national saving is (a) 130. (b) 120. (c) 110. (d) 100. (e) 90. (f) 80. (g) 70. (h) 60. (i) 50. (j) 40. (k) 30. (l) 20. (m) 10. (n) 0.

<table>
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<th>Consumption</th>
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<td>Investment</td>
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<td>Government spending</td>
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<td>Net taxes</td>
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<td>Imports</td>
<td>30</td>
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<td>Exports</td>
<td>20</td>
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2. If money wages rise the instant that the money supply increases, the price level (a) and output both would rise. (b) would rise and output would fall. (c) would rise and output would remain constant. (d) and output both would fall. (e) and output both would remain constant.

3. A Lorenz curve plots the cumulative percentage of income against the cumulative percentage of (a) area. (b) output. (c) consumption. (d) population.

4. The policy tool of the central bank that relies on bank borrowing is the (a) required reserve ratio for depository institutions. (b) tax rate on bank profits. (c) buying and selling of government bonds. (d) discount rate.

5. A business cycle shock is the (a) economic event that begins a business cycle fluctuations. (b) public response to economic fluctuations. (c) mechanism that causes a cycle to continue. (d) effects of an economic impulse to be eliminated.

6. Intertemporal substitution in labor supply describes shifts over time in labor supply in response to changes in (a) money wages. (b) real wages. (c) the real interest rate. (d) consumer demand for goods. (e) the real income.

7. Mr. Wang works as a clerk in a retail store during the day and is taking accounting classes in the evening. Mr. Wang is (a) not producing a good or a service. (b) producing service. (c) producing outside his production possibility frontier. (d) accumulating human capital.
(請注意: 第 8 - 10 小題為多重選項)

8. If the real interest rate is below the equilibrium point for the demand for investment funds and the supply of funds, (a) lenders will be unable to find borrowers willing to borrow all of the available funds and the real interest rate will fall. (b) borrowers will be unable to borrow all of the funds they want to borrow and the real interest rate will rise. (c) lenders will be unable to find borrowers willing to borrow all of the available funds and the real interest rate will rise. (d) borrowers will be unable to borrow all of the funds they want to borrow and the real interest rate will fall. (e) a rationing scheme may arise for the funds to be rationed. (f) the price mechanism will cause the real interest rate to rise.

9. Without international borrowing and lending, with free trade a country’s exports will (a) be less than its imports when the money supply increases. (b) exceed its imports when the money supply increases. (c) equal its imports independent of the money supply. (d) equal its output net of domestic expenditures. (e) be less than its output net of domestic expenditures when the money supply increases.

10. All of the following are necessary preconditions for economic growth except (a) restrictions on private ownership of factors of production. (b) markets. (c) property rights. (d) exchange using goods rather than money. (e) monetary exchange.

二、解釋下列名詞, 並說明該名詞之概念在總體經濟理論中的意義: (20%, 每小題各 10 分)

1. Lucas 的批評 (Lucas critique)
2. 效率工資 (efficiency wage)

三、假設中央銀行能以貨幣政策完全操控通貨膨脹率, 該央行所欲解決之最適化政策問題如下:

\[
\begin{align*}
\max_{\pi} & \quad U(u, \pi) = -(u + \alpha \pi^2), \quad \alpha > 0 \\
\text{s.t.} & \quad \pi = \beta(u - u_0) + \pi^e, \quad \beta > 0, \quad u_0 > 0
\end{align*}
\]

上述問題中, 式 (1) 爲央行之「目標函數」, 受限於式 (2) 所代表之「引進預期的 Phillips 曲線」, 假設央行決定採行權衡性的貨幣政策, 請您代央行當局計算並回答下列問題:

1. 民眾事先沒有預期物價上漲的心理(即 \(\pi^e = 0\)), 央行的最佳通貨膨脹率為何? (10%)
2. 民眾有事先完全預知(perfect foresight) 通貨膨脹率的能力(即 \(\pi^e = \pi\)), 央行的最佳通貨膨脹率為何? (10%)
3. 比較上述 1 與 2 兩種情況下所達成的社會福利水準之高低, 並說明此種結果之原因。 (10%)

第三題之符號說明:

\(u\) = 失業率; \(u_0\) = 自然失業率(國定); \(\pi\) = 通貨膨脹率; \(\pi^e\) = 預期通貨膨脹率;
\(U(u, \pi)\) = 中央銀行所設定為政策目標之社會福利函數。