總分100分，請依序作答。

1. (15%) Show that the point (2,4) lies on the curve \( x^3 + y^3 - 9xy = 0 \). Then find the tangent and normal to the curve on point (2,4).

![Graph of the curve](image)

2. (15%) Find the derivative \( \frac{dy}{dx} \) of the following functions:
   
a. \( y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}} \), \( x > 2 \)
   
b. \( y = \log_5(3x - 7) \), \( x > \frac{7}{3} \)
   
c. \( y^2 = 2e^{-\frac{1}{x}} \), \( x \neq 0 \)

3. (12%) Radioactive isotopes are often characterized by their half-life. Fill each of the four blanks (i)–(iv) in the table below. Carefully show step-by-step algebra that justifies your answers for (i)–(iv), as indicated.

<table>
<thead>
<tr>
<th>ISOTOPE</th>
<th>Half-Life (year)</th>
<th>Initial Amount (gram)</th>
<th>Amount after 1000 years (gram)</th>
<th>Amount after 10000 years (gram)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C^{14}</td>
<td>5730</td>
<td>3</td>
<td>(i)</td>
<td>(ii)</td>
</tr>
<tr>
<td>Ra^{226}</td>
<td>1620</td>
<td>(iii)</td>
<td>(iv)</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\[ e = 2.718 \]
\[ \ln 2 = 0.693 \]

4. (15%) Let \( h(x) \) be the function graphed below, and suppose that the shaded region has area equal to 1. Find the average value of \( h(x) \) over the interval \(-2 \leq x \leq 3\).

![Graph of the function](image)
5. (18%) Find the derivative \( \frac{dy}{dx} \) of the following functions:

a. \[ y = \frac{1}{x} (x^2 + e^x) \]

b. \[ y^2 = x^2 + \sin(xy) \]

c. \[ y = x^x, \ x > 0 \]

d. \[ y = \sin^{-1} x^2 \]

e. \[ y = x(\ln x)^2 \]

f. \[ x^y = y^x, x > 0, y > 0 \]

6. (25%) Compute:

a. \[ \lim_{x \to 0} \frac{x \sin(x)}{\cos(x) - 1} = \quad \]

b. \[ \int_{0}^{\pi} \sin(x) \, dx = \quad \]

c. \[ \frac{d}{dx} \sin(e^x) = \quad \]

d. \[ \int (x^2 + 3x + 5) \, dx = \quad \]

e. \[ \int_{-\infty}^{0} \theta e^\theta \, d\theta = \quad \]
INSTRUCTIONS:

1. If any question is unclear or you believe some assumptions need to be made, state your assumptions clearly at the beginning of your answer.

2. Unless stated otherwise, all the line numbers are for reference only.

1. (15%) What would be the output of the following C program?

```c
#include <stdio.h>
int f(int i)
{
    static int v = i;
    int t = v;
    v += i;
    return t;
}

int main()
{
    int i;
    for (i = 1; i <= 8; i <<= 1)
        printf("%d
", f(i));
    return 0;
}
```

2. (10%) What would be the output of the following C program?

```c
#include <stdio.h>
int f(int i)
{
    int result = -1;
    switch (i) {
    default:
        return 0;
    case 1:
        result = 1; break;
    case 2:
        result = 2;
    case 3:
        result = 3;
    }
    return result;
}

int main()
{
    int i;
    for (i = 1; i <= 4; i++)
        printf("%d
", f(i));
    return 0;
}
```
3. (15%) Use `typedef` to declare “foo” as an array of 3 pointers to functions taking as input an integer and returning as output an integer in a single declaration in C.

4. (10%) Define an integer variable “a” in C so that it can only be seen in the file at which it is defined.

5. (15%) Define in a single declaration in C a pointer to integer “p” so that p[1] is an alias of a[0], p[2] is an alias of a[1], and so on, all the way up to p[128] is an alias of a[127] for the integer array “int a[128];” defined in C.

6. (10%) What is (most) wrong with the C function f below?

```c
static int i = -1;
int *f()
{
  int a[10];
  for (i = 0; i < 10; i++)
  {
    a[i] = i;
  }
  return a;
}
```

7. (15%) What would be the output of the following C program?

```c
#include <stdio.h>
#define SQUARE(x) x*x
int main()
{
  printf("%d\n", SQUARE(1+2*3)*SQUARE(3+4*5));
  return 0;
}
```

8. (10%) What is (most) wrong with the C function g below?

```c
#include <stdlib.h>
#include <string.h>
char *g()
{
  char *d = (char *) malloc(128);
  char *s = "Now is the time...
  strcpy(d, s);
  free(s);
  return d;
}
```